

Chapter 2: Analytic Geometry: Line Segments and Circles

Getting Started, page 68

1. a) viii) The red line connects nonadjacent vertices of the parallelogram; so, it is a diagonal.
 b) vii) This triangle has all its sides of different lengths; so, it is scalene.
 c) ii) The red line intersects the blue one at a right angle, cutting it into two equal halves. Therefore, it is a perpendicular bisector.
 d) v) The red line joins the midpoint of one side of a triangle to the opposite vertex; so, it is a median.
 e) iv) The red line joins the midpoints of two sides of the triangle; so, it is a midsegment.
 f) vi) This grid has an x-axis and a y-axis marked with a scale; so, it is a Cartesian coordinate system.
 g) i) This triangle has exactly two sides of equal length; so, it is an isosceles triangle.
 h) iii) This triangle has all three sides of equal length; so, it is an equilateral triangle.

2. a) By the Pythagorean theorem:

$$x^2 = 5^2 + 12^2$$

$$x^2 = 25 + 144$$

$$x^2 = 169$$

$$x = 13$$

The length of side x is 13 m.

b) By the Pythagorean theorem:

$$x^2 = 200^2 - 55^2$$

$$x^2 = 40\,000 - 3\,025$$

$$x^2 = 36\,975$$

$$x \approx 192.3$$

The length of side x is about 192.3 mm.

3. a) Solve for the slope. Substitute a point into the equation to solve for the y-intercept.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{7 - 3}{7 - (-5)}$$

$$= \frac{4}{12}$$

$$= \frac{1}{3}$$

$$y = \frac{1}{3}x + b$$

$$7 = \frac{1}{3}(7) + b$$

$$b = 7 - \frac{7}{3}$$

$$b = \frac{14}{3}$$

Therefore, the equation of the line is $y = \frac{1}{3}x + \frac{14}{3}$.

- b) The slope of the perpendicular line is the negative reciprocal of the slope of the given equation, or -4 . Substitute the point into the equation to solve for the y-intercept.

$$y = -4x + b$$

$$-2 = -4(-1) + b$$

$$b = -2 - 4$$

$$b = -6$$

Therefore, the equation of the line is $y = -4x - 6$.

- c) The slope of the parallel line is the same as the slope of the given equation. Substitute the point into the equation to solve for the y-intercept.

$$y = -5x + b$$

$$-3 = -5(4) + b$$

$$b = -3 + 20$$

$$b = 17$$

Therefore, the equation of the line is $y = -5x + 17$.

$$4. a) \frac{1}{2}(-6) + \frac{3}{2} = -\frac{6}{2} + \frac{3}{2} = -\frac{3}{2}$$

$$b) \frac{3}{8} - \frac{3}{7} = \frac{21}{56} - \frac{24}{56} = -\frac{3}{56}$$

$$c) \frac{2}{3}x + 11x = \frac{2}{3}x + \frac{33}{3}x = \frac{35}{3}x$$

$$d) \frac{3}{4}y - \frac{3}{8}y = \frac{6}{8}y - \frac{3}{8}y = \frac{3}{8}y$$

$$e) \frac{2}{3}\left(\frac{3}{5}\right) + \frac{3}{4} = \frac{6}{15} + \frac{3}{4} = \frac{24}{60} + \frac{45}{60} = \frac{69}{60} = \frac{23}{20}$$

$$f) -1.5(0.625) + 4(-0.125) = -0.9375 - 0.5 = -1.4375$$

$$5. a) 3(7 - 4x) - \frac{4}{3}(2x + 1) = 49$$

$$21 - 12x - \frac{8}{3}x - \frac{4}{3} = 49$$

$$-\frac{36}{3}x - \frac{8}{3}x = \frac{147}{3} + \frac{4}{3} - \frac{63}{3}$$

$$-44x = 88$$

$$x = -2$$

$$b) \frac{1}{4}(x + 3) + \frac{1}{3}(x - 2) = -\frac{1}{2}$$

$$3(x + 3) + 4(x - 2) = -6$$

$$3x + 9 + 4x - 8 = -6$$

$$7x = -7$$

$$x = -1$$

$$\begin{aligned} \text{c) } \frac{x+4}{4} - \frac{x-2}{3} &= 1 \\ 3(x+4) - 4(x-2) &= 12 \\ 3x+12-4x+8 &= 12 \\ -x+20 &= 12 \\ -x &= -8 \\ x &= 8 \end{aligned}$$

$$\text{d) } x^2 = 36$$

$$x = 6 \text{ or } -6$$

$$\text{e) } x^2 + 16 = 25$$

$$x^2 = 9$$

$$x = 3 \text{ or } -3$$

$$\text{f) } 225 + x^2 = 289$$

$$x^2 = 64$$

$$x = 8 \text{ or } -8$$

6. a) Set the two equations equal to one another.

$$2x + 5 = 3x + 4$$

$$x = 1$$

Therefore, the two lines intersect at (1, 7).

b) The equations are:

$$(1) 4x + 2y = 7$$

$$(2) 6x - 4y = 0$$

Multiply equation (1) by 2.

$$(3) 8x + 4y = 14$$

Add equations (2) and (3) and solve for x.

$$14x = 14$$

$$x = 1$$

Substitute this value into equation (1) to solve for y.

$$4x + 2y = 7$$

$$4(1) + 2y = 7$$

$$2y = 3$$

$$y = \frac{3}{2}$$

Therefore, the lines intersect at $\left(1, \frac{3}{2}\right)$.

7. a) Mean

$$= [7 + (-11) + 23 + 5] \div 4$$

$$= 24 \div 4$$

$$= 6$$

$$\text{b) Mean} = \frac{\left(-\frac{1}{6} + \frac{2}{3}\right)}{2}$$

$$= \frac{\left(-\frac{1}{6} + \frac{4}{6}\right)}{2}$$

$$= \frac{\frac{3}{6}}{2}$$

$$= \frac{1}{4}$$

c) Mean

$$= [-1.4 + 3.6 + (-0.1)] \div 3$$

$$= 2.1 \div 3$$

$$= 0.7$$

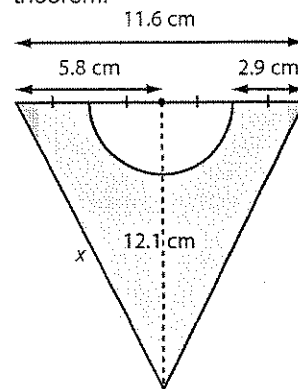
8. a) The radius of the large circle is 4.8 cm. The radii of the two smaller circles are each one half of this, or 2.4 cm. Calculate the area of the shaded region by

subtracting the areas of the two smaller circles from that of the larger.

$$\begin{aligned} A &= \pi R^2 - 2\pi r^2 \\ &= \pi(4.8)^2 - 2\pi(2.4)^2 \\ &= \pi(23.04) - 2\pi(5.76) \\ &= 72.38 - 36.19 \\ &= 36.2 \end{aligned}$$

The area is about 36.2 cm².

b) The radius of the semicircle is one-fourth of the base of the triangle, or 2.9 cm. Half of the base is 5.8 cm. The triangle is isosceles, so determine the lengths of the unknown sides by solving for x, using the Pythagorean theorem.



$$\begin{aligned} x^2 &= 12.1^2 + 5.8^2 \\ x^2 &= 146.41 + 33.64 \\ x^2 &= 180.005 \\ x &\approx 13.4 \end{aligned}$$

Calculate the perimeter of the shaded region by adding 2x, two one-fourth segments of the base, and the length of the curved side of the semi-circle.

$$\begin{aligned} P &= 2x + 2(b \div 4) + 2\pi r + 2 \\ &= 2(13.4) + 2(2.9) + \pi(2.9) \\ &= 26.8 + 5.8 + 9.1 \\ &= 41.7 \end{aligned}$$

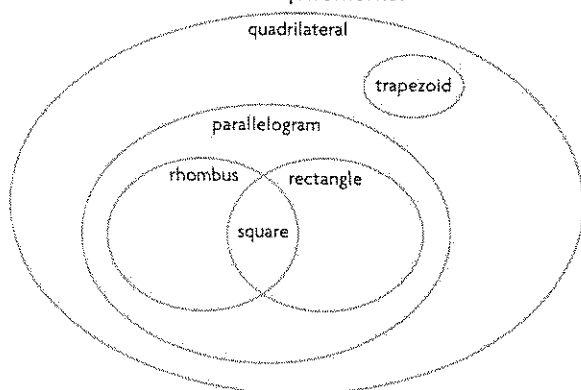
The perimeter is about 41.7 cm.

Calculate the area of the shaded region by subtracting the area of the semicircular region from the area of the triangle.

$$\begin{aligned} A &= bh \div 2 - \pi r^2 \div 2 \\ &= 11.6(12.1) \div 2 - \pi(2.9)^2 \div 2 \\ &= 11.6(12.1) \div 2 - 8.41\pi \div 2 \\ &= 70.18 - 13.21 \\ &= 57.0 \end{aligned}$$

The area is about 57.0 cm².

9. These are all four sided shapes, so they are all encompassed by the quadrilateral category. Parallelograms have parallel sides, whereas trapezoids do not, so these are separate groupings. Parallelograms include rectangles (which have 90° angles) and rhombuses (which have equal side lengths). A square is a parallelogram that is both a rectangle and a rhombus, since it meets all these requirements.



Lesson 2.1: Midpoint of a Line Segment, page 78

1. a)

$$\begin{aligned} M &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{1+5}{2}, \frac{1+9}{2} \right) \\ &= (3, 5) \end{aligned}$$

b)

$$\begin{aligned} M &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{-3+2}{2}, \frac{6+1}{2} \right) \\ &= (-0.5, 3.5) \end{aligned}$$

2. a)

$$\begin{aligned} M &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{-1+5}{2}, \frac{1+5}{2} \right) \\ &= (2, 3) \end{aligned}$$

c)

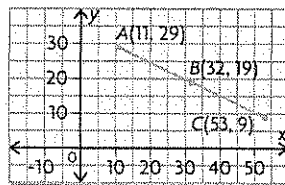
$$\begin{aligned} M &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{1+7}{2}, \frac{-2+(-2)}{2} \right) \\ &= (4, -2) \end{aligned}$$

b)

$$\begin{aligned} M &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{-2+3}{2}, \frac{6+(-4)}{2} \right) \\ &= (0.5, 1) \end{aligned}$$

3. a, b) The coordinates of the lamp are given using the midpoint formula.

$$\begin{aligned} M &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{11+53}{2}, \frac{29+9}{2} \right) \\ &= (32, 19) \end{aligned}$$



$$\begin{aligned} \text{4. a) } M &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{-1+5}{2}, \frac{3+7}{2} \right) \\ &= (2, 5) \end{aligned}$$

$$\begin{aligned} \text{b) } M &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{-2+3}{2}, \frac{3+4}{2} \right) \\ &= (0.5, 3.5) \end{aligned}$$

$$\begin{aligned} \text{c) } M &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{6+(-2)}{2}, \frac{-2+(-2)}{2} \right) \\ &= (2, -2) \end{aligned}$$

$$\begin{aligned} \text{d) } M &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{2+(-3)}{2}, \frac{-4+5}{2} \right) \\ &= (-0.5, 0.5) \end{aligned}$$

$$\begin{aligned} \text{e) } M &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{\frac{1}{2} + \left(-\frac{5}{2} \right)}{2}, \frac{-\frac{3}{2} + \left(-\frac{1}{2} \right)}{2} \right) \\ &= (-1, -1) \end{aligned}$$

$$\begin{aligned} \text{f) } M &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{1.5+(-1)}{2}, \frac{-2.5+4}{2} \right) \\ &= (0.25, 0.75) \end{aligned}$$

5. The centre of a circle is at the midpoint of its diameter. Use the midpoint formula.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left(\frac{-1 + 2.5}{2}, \frac{1 + (-3)}{2} \right)$$

$$= (0.75, -1)$$

6. The run from P to M is $1 - (-3) = 4$, and the rise is $1 - (-1) = 2$. The rise and run from M to Q must be the same, because M is the midpoint. Therefore, the x -coordinate of Q is $1 + 4 = 5$, and the y -coordinate is $1 + 2 = 3$.

7. a, b) Use the midpoint formula to determine the midpoints of each side.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$M_{AB} = \left(\frac{2 + (-4)}{2}, \frac{-2 + (-4)}{2} \right)$$

$$= (-1, -3)$$

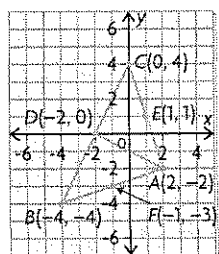
$$M_{BC} = \left(\frac{-4 + 0}{2}, \frac{-4 + 4}{2} \right)$$

$$= (-2, 0)$$

$$M_{AC} = \left(\frac{2 + 0}{2}, \frac{-2 + 4}{2} \right)$$

$$= (1, 1)$$

Let $D = M_{BC}$, $E = M_{AC}$, and $F = M_{AB}$. Draw a median from vertex A by connecting a line segment from point D to vertex A .



Use the formula to determine the slope of this median.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{0 - (-2)}{-2 - 2}$$

$$= -\frac{1}{2}$$

Then, the equation for the line is of the form:

$$y = -\frac{1}{2}x + b$$

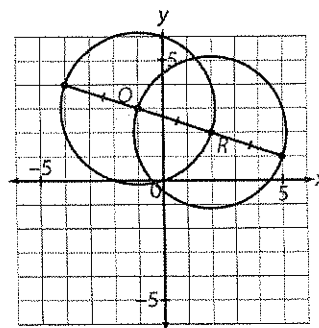
Substitute the coordinates of point D into this equation to solve for b .

$$0 = -\frac{1}{2}(-2) + b$$

$$b = -1$$

Therefore, the equation of this median is: $y = -\frac{1}{2}x - 1$.

8. For example, the centre of the circle could be assumed to be at O , or at R .



Assuming the centre is at O , the R must be one of the endpoints. The run from R to O is $-1 - 2 = -3$, and the rise is $3 - 2 = 1$. The rise and run from O to the other endpoint must be the same. Therefore, the other endpoint has an x -coordinate of $-1 - 3 = -4$, and a y -coordinate of $3 + 1 = 4$. So, the coordinates of the endpoints are $(-4, 4)$ and $(2, 2)$.

Assuming the centre is at R , then O must be one of the endpoints. The run from O to R is $2 - (-1) = 3$, and the rise is $2 - 3 = -1$. The rise and run from R to the other endpoint must be the same. Therefore, the other endpoint has an x -coordinate of $2 + 3 = 5$, and a y -coordinate of $2 - 1 = 1$. So, the coordinates of the endpoints are $(5, 1)$ and $(-1, 3)$.

9. The two diagonals are formed by connecting opposite vertices of the quadrilateral. The two pairs of opposite vertices are P and R , and Q and S in this case. Use the

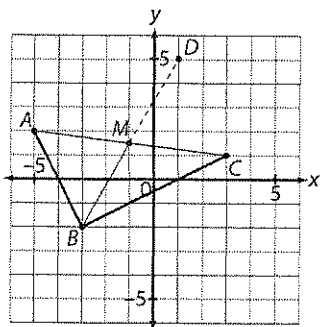
formula $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$ to determine the midpoints of each diagonal.

$$M_{PR} = \left(\frac{1 + 8}{2}, \frac{3 + 0}{2} \right) = \left(\frac{9}{2}, \frac{3}{2} \right)$$

$$M_{QS} = \left(\frac{6 + 3}{2}, \frac{5 + (-2)}{2} \right) = \left(\frac{9}{2}, \frac{3}{2} \right)$$

Indeed, the diagonals have the same midpoint.

10. In a rectangle, the diagonals bisect each other. This means that the midpoint of one diagonal is also the midpoint of the other diagonal. Mayda can determine the midpoint of the diagonal for which she knows the coordinates of both endpoints. Then she can use the coordinates of the midpoint and those of the known endpoint of the other diagonal to determine the missing coordinates. Here is a diagram of the process.



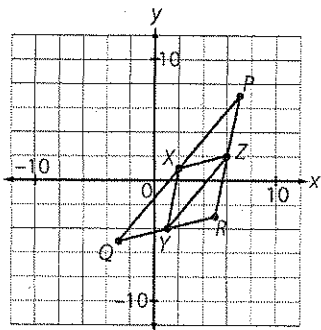
11. a) Use $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$ to determine the required midpoints.

$$M_{PQ} = \left(\frac{7 + (-3)}{2}, \frac{7 + (-5)}{2} \right) = (2, 1)$$

$$M_{QR} = \left(\frac{-3 + 5}{2}, \frac{-5 + (-3)}{2} \right) = (1, -4)$$

$$M_{PR} = \left(\frac{7 + 5}{2}, \frac{7 + (-3)}{2} \right) = (6, 2)$$

b) Let $X = M_{PQ}$, $Y = M_{QR}$, and $Z = M_{PR}$. The midsegments connect X and Y, Y and Z, and X and Z.



Use $m = \frac{y_2 - y_1}{x_2 - x_1}$ to determine the required slopes.

$$m_{XY} = \frac{-4 - 1}{1 - 2} = 5, \quad m_{YZ} = \frac{2 - (-4)}{6 - 1} = 1.2, \quad m_{XZ} = \frac{2 - 1}{6 - 2} = 0.25$$

c) Use the formula to calculate the slopes.

$$m_{PQ} = \frac{-5 - 7}{-3 - 7} = 1.2, \quad m_{QR} = \frac{-3 - (-5)}{5 - (-3)} = 0.25, \quad m_{PR} = \frac{-3 - 7}{5 - 7} = 5$$

d) For each midsegment, there is a side with the exact same slope. Thus, each midsegment is parallel to a side. Specifically, each midsegment is parallel to the side opposite it.

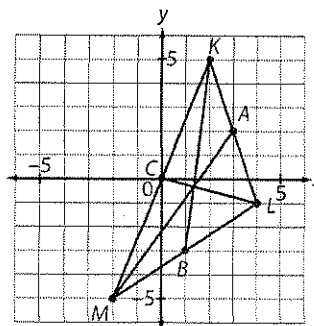
12. First, determine the midpoints of each side.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \quad M_{LM} = \left(\frac{4 + (-2)}{2}, \frac{-1 + (-5)}{2} \right) = (1, -3)$$

$$M_{KL} = \left(\frac{2 + 4}{2}, \frac{5 + (-1)}{2} \right) = (3, 2) \quad M_{KM} = \left(\frac{2 + (-2)}{2}, \frac{5 + (-5)}{2} \right) = (0, 0)$$

Let $A = M_{KL}$, $B = M_{LM}$, $C = M_{KM}$.

Medians therefore join points A and M, points B and K, and points C and L.



Determine the slopes of each of these medians using the

$$\text{formula } m = \frac{y_2 - y_1}{x_2 - x_1}.$$

$$m_{AM} = \frac{-5 - 2}{-2 - 3} = \frac{7}{5}, \quad m_{BK} = \frac{5 - (-3)}{2 - 1} = 8, \quad m_{CL} = \frac{-1 - 0}{4 - 0} = -\frac{1}{4}$$

Substitute a point (e.g. the relevant vertex) into each equation to solve for b .

For AM:	For BK:	For CL:
$y = m_{AM}x + b$	$y = m_{BK}x + b$	$y = m_{CL}x + b$
$-5 = \frac{7}{5}(-2) + b$	$5 = 8(2) + b$	$-1 = -\frac{1}{4}(4) + b$
$b = -\frac{11}{5}$	$b = -11$	$b = 0$

Therefore, the equation for the median AM is

$$y = \frac{7}{5}x - \frac{11}{5}, \text{ the equation for the median BK is}$$

$$y = 8x - 11, \text{ and the equation for the median CL is}$$

$$y = -\frac{1}{4}x.$$

13. a) First, bisect the line segment by finding the midpoint.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ = \left(\frac{-2 + 4}{2}, \frac{0 + (-4)}{2} \right) \\ = (1, -2)$$

The slope of the perpendicular bisector will be equal to the negative reciprocal of this slope, or $\frac{3}{2}$. Substitute the value of the midpoint into the equation for a line to solve for the y-intercept.

$$y = mx + b \\ -2 = \frac{3}{2}(1) + b \\ b = -\frac{7}{2}$$

The equation of the perpendicular bisector is

$$y = \frac{3}{2}x - \frac{7}{2}$$

b) First, bisect the line segment by finding the midpoint.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ = \left(\frac{4 + 12}{2}, \frac{6 + (-4)}{2} \right) \\ = (8, 1)$$

The slope of the perpendicular bisector will be equal to the negative reciprocal of this slope, or $\frac{4}{5}$.

The equation of the perpendicular bisector is

$$y = \frac{4}{5}x - \frac{27}{5}$$

c) First, bisect the line segment by finding the midpoint.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ = \left(\frac{-2 + 8}{2}, \frac{-4 + 4}{2} \right) \\ = (3, 0)$$

Next, determine the slope of the line segment.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \\ = \frac{-4 - 0}{4 - (-2)} \\ = -\frac{2}{3}$$

Next, determine the slope of the line segment.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \\ = \frac{-4 - 6}{12 - 4} \\ = -\frac{5}{4}$$

Substitute the value of the midpoint into the equation for a line to solve for the y-intercept.

$$y = mx + b \\ 1 = \frac{4}{5}(8) + b \\ b = -\frac{27}{5}$$

Next, determine the slope of the line segment.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \\ = \frac{4 - (-4)}{8 - (-2)} \\ = \frac{4}{5}$$

The slope of the perpendicular bisector will be equal to the negative reciprocal of this slope, or $-\frac{5}{4}$.

The equation of the perpendicular bisector is

$$y = -\frac{5}{4}x + \frac{15}{4}$$

d) First, bisect the line segment by finding the midpoint.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ = \left(\frac{-5 + 1}{2}, \frac{6 + (-2)}{2} \right) \\ = (-2, 2)$$

The slope of the perpendicular bisector will be equal to the negative reciprocal of this slope, or $\frac{3}{4}$.

The equation of the perpendicular bisector is

$$y = \frac{3}{4}x + \frac{7}{2}$$

14. Draw a diagram with a line connecting the two towns. The site could be located at any point on the perpendicular bisector to this line segment, as any point on the bisector will be equidistant from the towns.

Determine the midpoint of the segment.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ = \left(\frac{3 + 13}{2}, \frac{10 + 4}{2} \right) \\ = (8, 7)$$

Substitute the value of the midpoint into the equation for a line to solve for the y-intercept.

$$y = mx + b \\ 0 = \frac{-5}{4}(3) + b \\ b = \frac{15}{4}$$

Next, determine the slope of the line segment

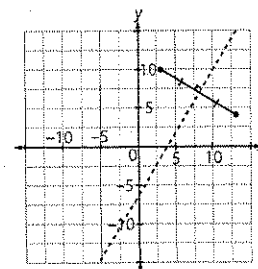
$$m = \frac{y_2 - y_1}{x_2 - x_1} \\ = \frac{-2 - 6}{1 - (-5)} \\ = -\frac{4}{3}$$

Substitute the value of the midpoint into the equation for a line to solve for the y-intercept.

$$y = mx + b \\ 2 = \frac{3}{4}(-2) + b \\ b = \frac{7}{2}$$

Next, determine the slope of the line segment.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \\ = \frac{4 - 10}{13 - 3} \\ = -\frac{3}{5}$$



The slope of the perpendicular bisector will be equal to the negative reciprocal of this slope, or $\frac{5}{3}$.

Substitute the value of the midpoint into the equation for a line to solve for the y-intercept.

$$y = mx + b$$

$$7 = \frac{5}{3}(8) + b$$

$$b = -\frac{19}{3}$$

The equation of the perpendicular bisector is

$$y = \frac{5}{3}x - \frac{19}{3}$$

15. Use the formula $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$ to

determine the midpoints of each side.

$$M_{DE} = \left(\frac{8 + (-4)}{2}, \frac{7 + 1}{2} \right)$$

$$= (2, 4)$$

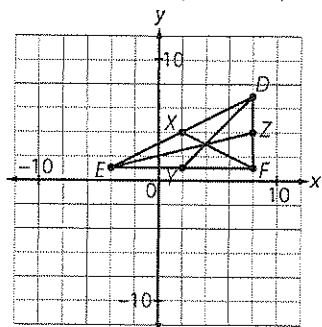
$$M_{EF} = \left(\frac{-4 + 8}{2}, \frac{1 + 1}{2} \right)$$

$$= (2, 1)$$

$$M_{DF} = \left(\frac{8 + 8}{2}, \frac{7 + 1}{2} \right)$$

$$= (8, 4)$$

Let $X = M_{DE}$, $Y = M_{EF}$, and $Z = M_{DF}$. Thus, the medians connect X and F , Y and D , and Z and E .



Use $m = \frac{y_2 - y_1}{x_2 - x_1}$ to determine the slope of each median.

$$m_{XF} = \frac{4 - 1}{2 - 8} = -\frac{1}{2}$$

$$m_{YD} = \frac{1 - 7}{2 - 8} = 1$$

$$m_{ZE} = \frac{4 - 1}{8 - (-4)} = \frac{1}{4}$$

Substitute a point into the equation of each median to solve for b .

For XF :

$$y = m_{XF}x + b$$

$$1 = -\frac{1}{2}(8) + b$$

$$b = 5$$

$$y = -\frac{1}{2}x + 5$$

For YD :

$$y = m_{YD}x + b$$

$$7 = 1(8) + b$$

$$b = -1$$

$$y = x - 1$$

For ZE :

$$y = m_{ZE}x + b$$

$$1 = \frac{1}{4}(-4) + b$$

$$b = 2$$

$$y = \frac{1}{4}x + 2$$

Substitute to determine the point of intersection between XF and YD . Set the two equations equal to one another and solve for x .

$$-\frac{1}{2}x + 5 = x - 1$$

$$\frac{3}{2}x = 6$$

$$x = 4$$

Substitute this value into the equation for YD to solve for y .

$$y = x - 1$$

$$y = 4 - 1$$

$$y = 3$$

Therefore, XF and YD intersect at $(4, 3)$.

Use substitution to determine the point of intersection between YD and ZE . Set the two equations equal to one another and solve for x .

$$\frac{1}{4}x + 2 = x - 1$$

$$\frac{3}{4}x = 3$$

$$x = 4$$

Substitute this value into the equation for YD to solve for y .

$$y = x - 1$$

$$y = 4 - 1$$

$$y = 3$$

Therefore, YD and ZE intersect at $(4, 3)$. But this is the same point where XF and YD intersect. Therefore, all the medians of this triangle intersect at $(4, 3)$. This point is called the centroid of the triangle.

16. a) The graph appears to have an x-intercept at $(1, 0)$, and a y-intercept at $(0, -3)$. Therefore, using the slope formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-3 - 0}{0 - 1}$$

$$= 3$$

The y-intercept of the graph is -3 , so the equation of the line is $y = 3x - 3$.

b) First, determine the coordinates of all points from inspection of the graph. The coordinates are: $A(5, 2)$, $B(9, 4)$, $C(6, -5)$, $A'(-1, 4)$, $B'(-3, 8)$, $C'(-6, -1)$. Next, determine the midpoint of a segment.

$$M_{AA'} = \left(\frac{5 + (-1)}{2}, \frac{2 + 4}{2} \right)$$

$$= (2, 3)$$

$$M_{BB'} = \left(\frac{9 + (-3)}{2}, \frac{4 + 8}{2} \right)$$

$$= (3, 6)$$

$$M_{CC'} = \left(\frac{6 + (-6)}{2}, \frac{-5 + (-1)}{2} \right)$$

$$= (0, -3)$$

Determine the slope of each of these segments using the

formula $m = \frac{y_2 - y_1}{x_2 - x_1}$.

$$m_{AA'} = \frac{2-4}{5-(-1)} \quad m_{BB'} = \frac{4-8}{9-(-3)} \quad m_{CC'} = \frac{-5-(-1)}{6-(-6)}$$

$$= -\frac{1}{3} \quad = -\frac{1}{3} \quad = -\frac{1}{3}$$

Thus the slope of the perpendicular bisectors is 3. Substitute the coordinates of the midpoints of each bisector into the equations of lines with the above slope, to solve for the y-intercepts.

For AA' :

$$y = mx + b$$

$$3 = 3(2) + b$$

$$b = -3$$

The equation of the perpendicular bisector for AA' is $y = 3x - 3$.

For CC' :

$$y = mx + b$$

$$-3 = 3(0) + b$$

$$b = -3$$

The equation of the perpendicular bisector for CC' is $y = 3x - 3$.

c) The line of reflection and all the perpendicular bisectors have the same equation: $y = 3x - 3$. This is the case because, when a shape is reflected across a line of reflection, each point of that shape moves an equal perpendicular distance away from the line on the other side.

17. Use the formula $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$ to

determine the midpoints of each side.

$$M_{WX} = \left(\frac{-7 + (-3)}{2}, \frac{-4 + 1}{2} \right) \quad M_{XY} = \left(\frac{-3 + 4}{2}, \frac{1 + 2}{2} \right)$$

$$= \left(-5, -\frac{3}{2} \right) \quad = \left(\frac{1}{2}, \frac{3}{2} \right)$$

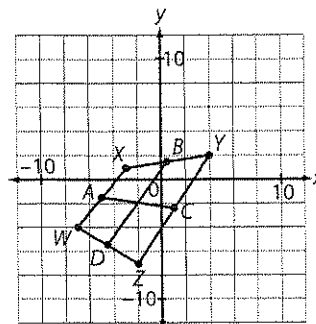
$$M_{YZ} = \left(\frac{4 + (-2)}{2}, \frac{2 + (-7)}{2} \right)$$

$$= \left(1, -\frac{5}{2} \right)$$

$$M_{ZW} = \left(\frac{-2 + (-7)}{2}, \frac{-7 + (-4)}{2} \right)$$

$$= \left(-\frac{9}{2}, -\frac{11}{2} \right)$$

Let $A = M_{WX}$, $B = M_{XY}$, $C = M_{YZ}$, $D = M_{ZW}$. Next, determine the slopes of the lines joining the midpoints of the non-adjacent sides. These lines are AC and BD .



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_{AC} = \frac{-\frac{3}{2} - \left(-\frac{5}{2}\right)}{-5 - 1}$$

$$= -\frac{1}{6}$$

$$m_{BD} = \frac{\frac{3}{2} - \left(-\frac{11}{2}\right)}{\frac{1}{2} - \left(-\frac{9}{2}\right)}$$

$$= \frac{7}{5}$$

Substitute one endpoint from each of the above two lines into their equations, to solve for the y-intercepts.

For AC :

$$y = m_{AC}x + b$$

$$-\frac{3}{2} = \left(-\frac{1}{6}\right)(-5) + b$$

$$b = -\frac{7}{3}$$

The equation for AC is

$$y = -\frac{1}{6}x - \frac{7}{3}$$

For BD :

$$y = m_{BD}x + b$$

$$\frac{3}{2} = \left(\frac{7}{5}\right)\left(\frac{1}{2}\right) + b$$

$$b = \frac{4}{5}$$

The equation for BD is

$$y = \frac{7}{5}x + \frac{4}{5}$$

Set the above two equations equal to one another to determine the point of intersection.

$$\frac{7}{5}x + \frac{4}{5} = -\frac{1}{6}x - \frac{7}{3}$$

$$42x + 24 = -5x - 70$$

$$47x = -94$$

$$x = -2$$

Substitute this value to solve for y .

$$y = \frac{7}{5}x + \frac{4}{5}$$

$$y = \frac{7}{5}(-2) + \frac{4}{5}$$

$$y = -2$$

Thus, the two lines intersect at $(-2, -2)$.

18. For example, i) Using the midpoint formula:

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

ii) Using rise and run: Starting at one point, determine the rise and run. Then add half the run to the x-coordinate of this point and add half the rise to the y-coordinate of this point. These strategies are similar because they give the same answer, but are different because in i) you are using the mean value of x and y but in ii) you are using the difference between x and y.

19. Determine the rise and run from A to B.

$$\text{Run} = 10 - 1$$

$$= 9$$

$$\text{Rise} = 4 - 7$$

$$= -3$$

Add one-third of the run to the x-coordinate of A, and one-third of the rise to the y-coordinate of A.

$$x = 1 + (9 \div 3)$$

$$x = 4$$

$$y = 7 + (-3 \div 3)$$

$$y = 6$$

Therefore, this point has coordinates of (4, 6).

20. a) Determine M, the midpoint of TU.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left(\frac{-6 + 0}{2}, \frac{12 + (-12)}{2} \right)$$

$$= (-3, 0)$$

Determine the rise and run from S to M.

$$\text{Run} = -3 - 6$$

$$= -9$$

$$\text{Rise} = 0 - 6$$

$$= -6$$

Add two-thirds of the run to the x-coordinate of S, and two-thirds of the rise to the y-coordinate of S.

$$x = 6 + 2(-9) \div 3$$

$$x = 0$$

$$y = 6 + 2(-6) \div 3$$

$$y = 2$$

Therefore, this point lies at (0, 2).

b) For TN: Determine N, the midpoint of SU.

$$N = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left(\frac{6 + 0}{2}, \frac{6 + (-12)}{2} \right)$$

$$= (3, -3)$$

Determine the rise and run from T to N.

$$\text{Run} = 3 - (-6)$$

$$= 9$$

$$\text{Rise} = -3 - 12$$

$$= -15$$

Add two-thirds of the run to the x-coordinate of T, and two-thirds of the rise to the y-coordinate of T.

$$x = -6 + 29 \div 3$$

$$x = 0$$

$$y = 12 + 2(-15) \div 3$$

$$y = 2$$

Therefore, this point lies at (0, 2).

For UR: Determine R, the midpoint of ST.

$$R = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left(\frac{6 + (-6)}{2}, \frac{6 + 12}{2} \right)$$

$$= (0, 9)$$

Determine the rise and run from U to R.

$$\text{Run} = 0 - 0$$

$$= -9$$

$$\text{Rise} = 9 - (-12)$$

$$= 21$$

Add two-thirds of the run to the x-coordinate of U, and two-thirds of the rise to the y-coordinate of U.

$$x = 0 + 2(-9) \div 3$$

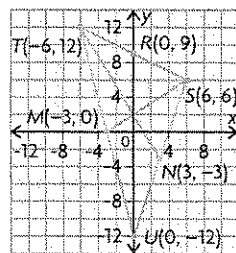
$$x = 0$$

$$y = -12 + 2(21) \div 3$$

$$y = 2$$

Therefore, this point lies at (0, 2).

c) Drawing the triangle and medians on a coordinate grid reveals that all the medians intersect at (0, 2).



Notice that this is the exact same point as the points that were found in parts a) and b). The three medians all intersect at a point that is two-thirds down the length of any median from its respective vertex.

d) For example, yes, because it seems to be true upon inspecting other triangles. Also, there is nothing special about this triangle to suggest this is a special case.

Lesson 2.2: Length of a Line Segment, page 86

1. Use the distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

$$\text{a) } AB = \sqrt{(5 - 1)^2 + (4 - 1)^2}$$

$$= \sqrt{4^2 + 3^2}$$

$$= \sqrt{16 + 9}$$

$$= \sqrt{25}$$

$$= 5$$

$$\text{b) } CD = 5 - (-1)$$

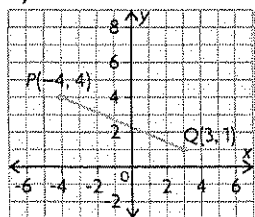
$$= 6$$

$$\begin{aligned}
 \text{c) } EF &= \sqrt{[3 - (-2)]^2 + (2 - 6)^2} \\
 &= \sqrt{5^2 + (-4)^2} \\
 &= \sqrt{25 + 16} \\
 &= \sqrt{41} \\
 &\approx 6.4
 \end{aligned}$$

$$\begin{aligned}
 \text{d) } GH &= 7 - (-3) \\
 &= 10
 \end{aligned}$$

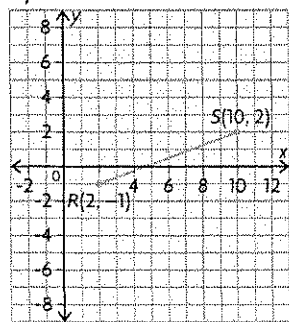
2. Use the distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

a)



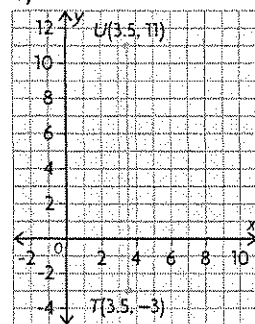
$$\begin{aligned}
 PQ &= \sqrt{[3 - (-4)]^2 + (1 - 4)^2} \\
 &= \sqrt{7^2 + (-3)^2} \\
 &= \sqrt{49 + 9} \\
 &= \sqrt{58} \\
 &\approx 7.6
 \end{aligned}$$

b)



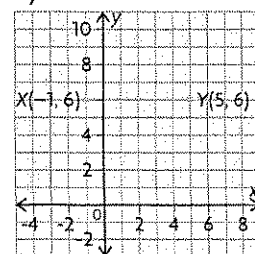
$$\begin{aligned}
 RS &= \sqrt{(10 - 2)^2 + [2 - (-1)]^2} \\
 &= \sqrt{8^2 + 3^2} \\
 &= \sqrt{64 + 9} \\
 &= \sqrt{73} \\
 &\approx 8.5
 \end{aligned}$$

c)



$$\begin{aligned}
 TU &= 11 - (-3) \\
 &= 14
 \end{aligned}$$

d)



$$\begin{aligned}
 XY &= 5 - (-1) \\
 &= 6
 \end{aligned}$$

3. a) Use the distance formula,

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, to determine the distance between the two locations.

$$\begin{aligned}
 KN &= \sqrt{[220 - (-70)]^2 + (490 - 770)^2} \\
 &= \sqrt{290^2 + (-280)^2} \\
 &= \sqrt{84\,100 + 78\,400} \\
 &= \sqrt{162\,500} \\
 &\approx 403
 \end{aligned}$$

The helicopter travelled about 403 km.

b) I assumed that the helicopter flew in a straight line between the two locations.

4. a) P is at point (-4, 4), and Q is at point (-4, 1). They have the same x-coordinate, so the distance between the two is

$$\begin{aligned}
 PQ &= 4 - 1 \\
 &= 3
 \end{aligned}$$

Use the distance formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, in parts b) to e).

b) Q is at point (-4, 1), and R is at point (2, 3).

$$\begin{aligned}
 QR &= \sqrt{[2 - (-4)]^2 + (3 - 1)^2} \\
 &= \sqrt{6^2 + 2^2} \\
 &= \sqrt{36 + 4} \\
 &= \sqrt{40} \\
 &\approx 6.3
 \end{aligned}$$

c) U is at point (-2, -4), and S is at point (4, -3).

$$\begin{aligned}
 US &= \sqrt{[4 - (-2)]^2 + [-3 - (-2)]^2} \\
 &= \sqrt{6^2 + (-1)^2} \\
 &= \sqrt{36 + 1} \\
 &= \sqrt{37} \\
 &\approx 6.1
 \end{aligned}$$

d) P is at point (-4, -4), and R is at point (2, 3).

$$\begin{aligned}
 PR &= \sqrt{[2 - (-4)]^2 + (3 - (-4))^2} \\
 &= \sqrt{6^2 + (-1)^2} \\
 &= \sqrt{36 + 1} \\
 &= \sqrt{37} \\
 &\approx 6.1
 \end{aligned}$$

e) P is at point (-4, -4), and U is at point (-2, -4).

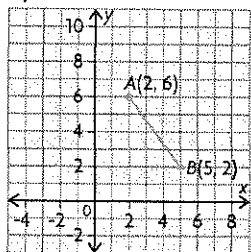
$$\begin{aligned}
 PU &= \sqrt{[-2 - (-4)]^2 + (-4 - (-4))^2} \\
 &= \sqrt{2^2 + (-8)^2} \\
 &= \sqrt{4 + 64} \\
 &= \sqrt{68} \\
 &\approx 8.2
 \end{aligned}$$

f) Q is at point $(-4, 1)$, and T is at point $(4, 1)$. They have the same y-coordinate, so the distance between the two is:

$$QT = 4 - (-4) = 8$$

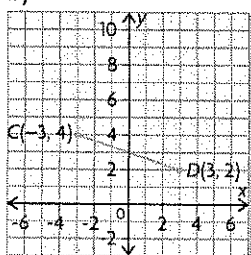
5. Use the formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ in parts a), b), d), and e).

a)



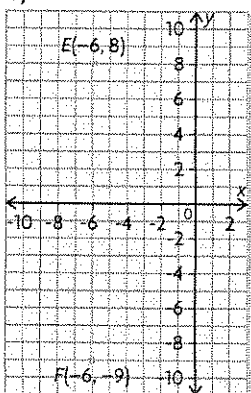
$$\begin{aligned} AB &= \sqrt{(2-6)^2 + (5-2)^2} \\ &= \sqrt{(-4)^2 + 3^2} \\ &= \sqrt{16+9} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

b)



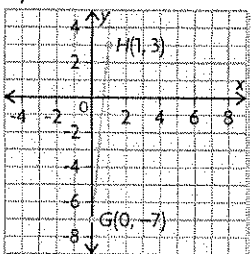
$$\begin{aligned} CD &= \sqrt{[3 - (-3)]^2 + (2-4)^2} \\ &= \sqrt{6^2 + (-2)^2} \\ &= \sqrt{36+4} \\ &= \sqrt{40} \\ &\approx 6.3 \end{aligned}$$

c)



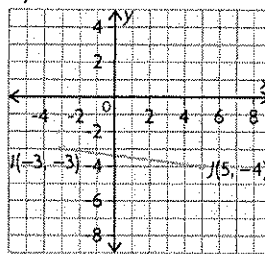
$$\begin{aligned} EF &= 8 - (-9) \\ &= 17 \end{aligned}$$

d)



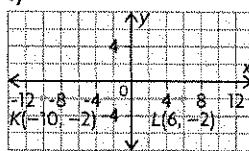
$$\begin{aligned} GH &= \sqrt{(1-0)^2 + [3 - (-7)]^2} \\ &= \sqrt{1^2 + 10^2} \\ &= \sqrt{1+100} \\ &= \sqrt{101} \\ &\approx 10.0 \end{aligned}$$

e)



$$\begin{aligned} IJ &= \sqrt{[5 - (-3)]^2 + [-4 - (-3)]^2} \\ &= \sqrt{8^2 + (-1)^2} \\ &= \sqrt{64+1} \\ &= \sqrt{65} \\ &\approx 8.1 \end{aligned}$$

f)



$$\begin{aligned} KL &= 6 - (-10) \\ &= 16 \end{aligned}$$

6. a) The line segment in part c) is vertical because the x-coordinates of the endpoints are the same. The line segment in part f) is horizontal because the y-coordinates of the endpoints are the same.

b) For a horizontal line segment, the length of the segment is equal to the positive difference between the x-coordinates of the endpoints. For a vertical line segment, the length of the segment is equal to the positive difference between the y-coordinates of the endpoints.

7. Use the formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to calculate the distance the ball travels from A to B.

$$\begin{aligned} AB &= \sqrt{(6-2)^2 + (5-3)^2} \\ &= \sqrt{4^2 + 2^2} \\ &= \sqrt{16+4} \\ &= \sqrt{20} \\ &\approx 4.47 \end{aligned}$$

Now calculate the distance the ball travels from B to C.

$$\begin{aligned} BC &= \sqrt{(2-6)^2 + (7-5)^2} \\ &= \sqrt{(-4)^2 + 2^2} \\ &= \sqrt{16+4} \\ &= \sqrt{20} \\ &\approx 4.47 \end{aligned}$$

Sum these two distances to determine the total distance that the ball travels.

$$\begin{aligned} \text{distance} &= AB + BC \\ &\approx 4.47 + 4.47 \\ &\approx 8.9 \end{aligned}$$

The ball travels approximately 8.9 units.

8. Use $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to calculate the distance from A to each point.

$$\begin{aligned} AB &= \sqrt{[1.8 - (-3.2)]^2 + (-4.3 - 5.6)^2} \\ &= \sqrt{5.0^2 + (-9.9)^2} \\ &= \sqrt{25.00 + 98.01} \\ &= \sqrt{123.01} \\ &\approx 11.1 \end{aligned}$$

$$\begin{aligned}
 AC &= \sqrt{[0.7 - (-3.2)]^2 + (8.9 - 5.6)^2} \\
 &= \sqrt{3.9^2 + 3.3^2} \\
 &= \sqrt{15.21 + 10.89} \\
 &= \sqrt{26.10} \\
 &\approx 5.1
 \end{aligned}$$

$$\begin{aligned}
 AD &= \sqrt{[-7.6 - (-3.2)]^2 + (3.9 - 5.6)^2} \\
 &= \sqrt{(-4.4)^2 + (-1.7)^2} \\
 &= \sqrt{19.36 + 2.89} \\
 &= \sqrt{22.25} \\
 &\approx 4.7
 \end{aligned}$$

Point D is closest to point A , at a distance of 4.7 units away.

9. Use $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to calculate the distance from each town to the blaze. Let F represent the fire, M represent Mordon, and B represent Bently.

$$\begin{aligned}
 FM &= \sqrt{(26 - 10)^2 + [77 - (-11)]^2} \\
 &= \sqrt{16^2 + 88^2} \\
 &= \sqrt{256 + 7744} \\
 &= \sqrt{8000} \\
 &\approx 89.4
 \end{aligned}$$

$$\begin{aligned}
 FB &= \sqrt{(12 - 10)^2 + [-88 - (-11)]^2} \\
 &= \sqrt{2^2 + (-77)^2} \\
 &= \sqrt{4 + 5929} \\
 &= \sqrt{5933} \\
 &\approx 77.0
 \end{aligned}$$

Therefore, Bently is closer to the fire, at about 77.0 units away.

10. Use $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to calculate the distance from point F to each point.

$$\begin{aligned}
 FA &= \sqrt{(22 - 1)^2 + [18 - (-1)]^2} \\
 &= \sqrt{21^2 + 19^2} \\
 &= \sqrt{441 + 361} \\
 &= \sqrt{802} \\
 &\approx 28.3
 \end{aligned}$$

$$\begin{aligned}
 FB &= \sqrt{(-3 - 1)^2 + [35 - (-1)]^2} \\
 &= \sqrt{(-4)^2 + 36^2} \\
 &= \sqrt{16 + 1296} \\
 &= \sqrt{1312} \\
 &\approx 36.2
 \end{aligned}$$

$$\begin{aligned}
 FC &= \sqrt{(7 - 1)^2 + [-29 - (-1)]^2} \\
 &= \sqrt{6^2 + (-28)^2} \\
 &= \sqrt{36 + 784} \\
 &= \sqrt{820} \\
 &\approx 28.6
 \end{aligned}$$

Beast ran the farthest, a distance of 36.2 units.

11. For example, they are the same because they both compare the x -values and y -values of the endpoints of the segment. They are different, because, while the midpoint formula calculates the average x -coordinates and average y -coordinates, the distance formula compares the differences between the x -coordinates for each endpoint, and the differences between the y -coordinates for each endpoint.

12. a) The shortest distance from a point to a line occurs along a line perpendicular to the given line. Determine the slope of a line perpendicular to the given line by taking the negative reciprocal of the given slope,

$$m = 4: -\frac{1}{4}$$

This line must pass through the given point; so, substitute this value into the equation to solve for the y -intercept.

$$\begin{aligned}
 y &= mx + b \\
 3 &= -\frac{1}{4}(-3) + b \\
 3 &= \frac{3}{4} + b \\
 b &= \frac{9}{4}
 \end{aligned}$$

Thus, this line has equation $y = -\frac{1}{4}x + \frac{9}{4}$.

Determine the point of intersection between this line and the given line.

$$y = -\frac{1}{4}x + \frac{9}{4}$$

Set the two equations equal to one another and solve for x .

$$\begin{aligned}
 4x - 2 &= -\frac{1}{4}x + \frac{9}{4} \\
 \frac{17}{4}x &= \frac{17}{4} \\
 x &= 1
 \end{aligned}$$

Substitute this value to solve for y .

$$\begin{aligned}
 y &= 4x - 2 \\
 y &= 4(1) - 2 \\
 y &= 2
 \end{aligned}$$

The closest point on the line from $(-3, 3)$ is $(1, 2)$. Use the formula to find the separation distance.

$$\begin{aligned}
 d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(-3 - 1)^2 + (3 - 2)^2} \\
 &= \sqrt{(-4)^2 + 1^2} \\
 &= \sqrt{16 + 1} \\
 &= \sqrt{17} \\
 &\approx 4.1
 \end{aligned}$$

The distance between the line and the point is 4.1 units.

b) The shortest distance from a point to a line occurs along a line perpendicular to the given line. Determine the slope of a line perpendicular to the given line by taking the negative reciprocal of the given slope, $m = -1$: 1.

This line must pass through the given point; so, substitute this value into the equation so far to solve for the y-intercept.

$$\begin{aligned} y &= mx + b \\ -2 &= 1(-1) + b \\ b &= -1 \end{aligned}$$

Thus, this line has equation $y = x - 1$.

Substitute this value into the given equation to solve for y.

$$\begin{aligned} y &= -x + 5 \\ y &= -(3) + 5 \\ y &= 2 \end{aligned}$$

The closest point on the line from $(-1, -2)$ is $(3, 2)$. Use the formula to find the separation distance.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-1 - 3)^2 + (-2 - 2)^2} \\ &= \sqrt{(-4)^2 + (-4)^2} \\ &= \sqrt{16 + 16} \\ &= \sqrt{32} \\ &\approx 5.7 \end{aligned}$$

The distance between the line and the point is about 5.7 units.

c) The shortest distance from a point to a line occurs along a line perpendicular to the given line. Determine the slope of a line that is perpendicular to the given line by taking the negative reciprocal of the slope of the given line.

This line must pass through the given point; so, substitute this value into the equation so far to solve for the y-intercept. Thus, this line has

$$\text{equation } y = \frac{3}{2}x - \frac{9}{2}$$

Determine the point of intersection between this line and the given line by substituting this equation into that of the given line, and solving for x.

Find the point of intersection between this line and the given line. Set the two equations equal to one another and solve for x.

$$\begin{aligned} x - 1 &= -x + 5 \\ 2x &= 6 \\ x &= 3 \end{aligned}$$

$$\begin{aligned} 2x + 3y &= 6 \\ 3y &= -2x + 6 \\ y &= -\frac{2}{3}x + 2 \\ m &= -\frac{2}{3} \end{aligned}$$

$$\begin{aligned} y &= mx + b \\ 6 &= \frac{3}{2}(7) + b \\ 6 &= \frac{21}{2} + b \\ b &= -\frac{9}{2} \\ 2x + 3\left(\frac{3}{2}x - \frac{9}{2}\right) &= 6 \\ 2x + \frac{9}{2}x - \frac{27}{2} &= 6 \\ \frac{13}{2}x &= \frac{39}{2} \\ x &= 3 \end{aligned}$$

Substitute this value into the equation for the perpendicular line to solve for y.

$$\begin{aligned} y &= \frac{3}{2}x - \frac{9}{2} \\ y &= \frac{3}{2}(3) - \frac{9}{2} \\ y &= \frac{9}{2} - \frac{9}{2} \\ y &= 0 \end{aligned}$$

The closest point on the line from $(7, 6)$ is $(3, 0)$. Use the formula to find the separation distance.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(7 - 3)^2 + (6 - 0)^2} \\ &= \sqrt{4^2 + 6^2} \\ &= \sqrt{16 + 36} \\ &= \sqrt{52} \\ &\approx 7.2 \end{aligned}$$

The distance between the line and the point is about 7.2 units.

d) The shortest distance from a point to a line is along a line perpendicular to the given line. Determine the slope of a line that is perpendicular to the given line by taking the negative reciprocal of the slope of the given line.

This line must pass through the given point; so, substitute this value into the equation so far to solve for the y-intercept.

Thus, this line has equation $y = -0.4x + 5.3$.

Determine the point of intersection between this line and the given line by substituting this equation into that of the given line, and solving for x.

Substitute this value into the equation for the perpendicular line to solve for y.

$(3.55, 3.88)$ is the closest point on the line from $(2, 4.5)$. Use the formula to find the separation distance.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(2 - 3.55)^2 + (4.5 - 3.88)^2} \\ &= \sqrt{(-1.55)^2 + (0.62)^2} \\ &= \sqrt{2.40 + 0.38} \\ &= \sqrt{2.78} \\ &\approx 1.7 \end{aligned}$$

The line and the point is about 1.7 units.

$$\begin{aligned} 5x - 2y &= 10 \\ -2y &= -5x + 10 \\ y &= \frac{5}{2}x - 5 \\ m &= \frac{5}{2} \\ y &= mx + b \\ 4.5 &= -0.4(2) + b \\ 4.5 &= -0.8 + b \\ b &= 5.3 \end{aligned}$$

$$\begin{aligned} 5x - 2(-0.4x + 5.3) &= 10 \\ 5x + 0.8x - 10.6 &= 10 \\ 5.8x &= 20.6 \\ x &\approx 3.55 \end{aligned}$$

$$\begin{aligned} y &= -0.4x + 5.3 \\ y &\approx -0.4(3.55) + 5.3 \\ y &\approx -1.42 + 5.3 \\ y &\approx 3.88 \end{aligned}$$

13. Assume that the roads travel towards the park in straight lines. Thus, building exits such that the roads are perpendicular to each highway will result in the shortest roads.

Highway 2: The slope of the perpendicular road is the negative reciprocal of the given slope, or -0.5 .

This road must lead to the park; so, substitute the coordinates of the park into the equation so far to determine the y -intercept.

$$\begin{aligned} y &= mx + b \\ 4 &= -0.5(3) + b \\ 4 &= -1.5 + b \\ b &= 5.5 \end{aligned}$$

Thus, the equation of the road from Highway 2 is $y = -0.5x + 5.5$. Determine the point where this road intersects the highway to determine the location for the exit.

$$\begin{aligned} -0.5x + 5.5 &= 2x + 5 \\ 2.5x &= 0.5 \\ x &= 0.2 \end{aligned}$$

Substitute this value into the equation for the highway to solve for y .

$$\begin{aligned} y &= 2x + 5 \\ y &= 2(0.2) + 5 \\ y &= 5.4 \end{aligned}$$

The exit for Highway 2 should be built at $(0.2, 5.4)$ or

$$\left(\frac{1}{5}, \frac{27}{5}\right).$$

Highway 10: The slope of the perpendicular road is the negative reciprocal of the given slope, or 2 .

This road must lead to the park; so, substitute the coordinates of the park into the equation so far to determine the y -intercept.

$$\begin{aligned} y &= mx + b \\ 4 &= 2(3) + b \\ 4 &= 6 + b \\ b &= -2 \end{aligned}$$

The equation of the road from Highway 10 is

$$y = 2x - 2.$$

Determine the point where this road intersects the highway to determine the location for the exit.

$$\begin{aligned} 2x - 2 &= -0.5x + 2 \\ 2.5x &= 4 \\ x &= 1.6 \end{aligned}$$

Substitute this value into the equation for the highway to solve for y .

$$\begin{aligned} y &= -0.5x + 2 \\ y &= -0.5(1.6) + 2 \\ y &= -0.8 + 2 \\ y &= 1.2 \end{aligned}$$

The exit for Highway 10 should be built at $(1.6, 1.2)$ or

$$\left(\frac{8}{5}, \frac{6}{5}\right).$$

14. First, calculate the distances between A and B , B and C , and C and D in grid-units, using

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ AB &= \sqrt{[-8 - (-18)]^2 + (1 - 12)^2} \\ &= \sqrt{10^2 + (-11)^2} \\ &= \sqrt{100 + 121} \\ &= \sqrt{221} \\ &\approx 14.87 \\ BC &= \sqrt{[3 - (-8)]^2 + (4 - 1)^2} \\ &= \sqrt{11^2 + 3^2} \\ &= \sqrt{121 + 9} \\ &= \sqrt{130} \\ &\approx 11.40 \\ CD &= \sqrt{(15 - 3)^2 + (7 - 4)^2} \\ &= \sqrt{12^2 + 3^2} \\ &= \sqrt{144 + 9} \\ &= \sqrt{153} \\ &\approx 12.37 \end{aligned}$$

Add to determine the total grid-units of cable required.

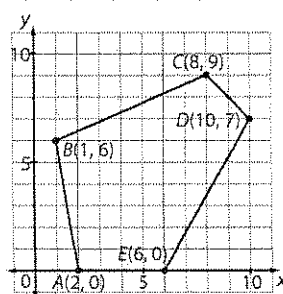
$$\begin{aligned} I &= AB + BC + CD \\ I &\approx 14.87 + 11.40 + 12.37 \\ I &\approx 38.64 \end{aligned}$$

Since one grid-unit is 2.5 m, multiply this distance by 2.5 m to determine the actual length of cable required.

$$\begin{aligned} \text{Length} &= 2.5I \\ &\approx 2.5(38.64) \\ &\approx 96.6 \end{aligned}$$

Therefore, about 96.6 m of cable are required.

15. Draw a diagram, labelling the points as: $A(2, 0)$, $B(1, 6)$, $C(8, 9)$, $D(10, 7)$, and $E(6, 0)$.



Determine the distances between each pair of adjacent points in grid units, using $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

$$\begin{aligned} AB &= \sqrt{(1 - 2)^2 + (6 - 0)^2} & BC &= \sqrt{(8 - 1)^2 + (9 - 6)^2} \\ &= \sqrt{(-1)^2 + 6^2} & &= \sqrt{7^2 + 3^2} \\ &= \sqrt{1 + 36} & &= \sqrt{49 + 9} \\ &= \sqrt{37} & &= \sqrt{58} \\ &\approx 6.08 & &\approx 7.62 \end{aligned}$$

$$CD = \sqrt{(10-8)^2 + (7-9)^2}$$

$$= \sqrt{2^2 + (-2)^2}$$

$$= \sqrt{4+4}$$

$$= \sqrt{8}$$

$$\approx 2.83$$

$$EA = \sqrt{(2-6)^2 + (0-0)^2}$$

$$= \sqrt{(-4)^2 + 0^2}$$

$$= \sqrt{16}$$

$$= 4$$

Sum the distances to find the total grid-units of fencing required.

$$I = AB + BC + CD + DE + EA$$

$$\approx 6.08 + 7.62 + 2.83 + 8.06 + 4$$

$$\approx 28.59$$

Since one grid-unit is 10 m, multiply this distance by 10 to determine the actual length of fencing required.

$$\text{Length} = 10I$$

$$= 10(28.59)$$

$$= 285.9$$

Therefore, about 285.9 m of fencing are required.

16. I would use the formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

to calculate the distance between each pair. For example, to determine whether A(3, 4) or B(-5, 2) is closer to C(1, 1):

$$AC = \sqrt{(1-3)^2 + (1-4)^2}$$

$$= \sqrt{(-2)^2 + (-3)^2}$$

$$= \sqrt{4+9}$$

$$= \sqrt{13}$$

$$\approx 3.6$$

$$BC = \sqrt{[1-(-5)]^2 + (1-2)^2}$$

$$= \sqrt{6^2 + (-1)^2}$$

$$= \sqrt{36+1}$$

$$= \sqrt{37}$$

$$\approx 6.1$$

Therefore, point A is closer.

17. a) If vertex A is translated such that it lies on the x-axis, then A' must have a y-coordinate of 0. A has a y-coordinate of 2, so the shift is $0 - 2 = -2$, or down 2. If vertex B is translated such that it lies on the y-axis, then B' must have an x-coordinate of 0. B has an x-coordinate of 4, so the shift is $0 - 4 = -4$, or left 4. So, the total translation is left 4, down 2. Apply this translation to the whole triangle to find the coordinates of triangle A'B'C'.

$$A'(1-4, 2-2) = A'(-3, 0), B'(4-4, 8-2) = B'(0, 6)$$

$$C'(8-4, 4-2) = C'(4, 2)$$

b) For example, congruent triangles have side identical side lengths. Determine the side lengths of each triangle

$$\text{using } d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

$$DE = \sqrt{(6-10)^2 + (0-7)^2}$$

$$= \sqrt{(-4)^2 + (-7)^2}$$

$$= \sqrt{16+49}$$

$$= \sqrt{65}$$

$$\approx 8.06$$

$\triangle ABC$:

$$AB = \sqrt{(4-1)^2 + (8-2)^2}$$

$$= \sqrt{3^2 + 6^2}$$

$$= \sqrt{9+36}$$

$$= \sqrt{45}$$

$$\approx 6.7$$

$$BC = \sqrt{(8-4)^2 + (4-8)^2}$$

$$= \sqrt{4^2 + (-4)^2}$$

$$= \sqrt{16+16}$$

$$= \sqrt{32}$$

$$\approx 5.7$$

$$CA = \sqrt{(1-8)^2 + (2-8)^2}$$

$$= \sqrt{(-7)^2 + (-6)^2}$$

$$= \sqrt{49+36}$$

$$= \sqrt{85}$$

$$\approx 9.2$$

$\triangle DEF$:

$$DE = \sqrt{[-2-(-1)]^2 + (6-1)^2}$$

$$= \sqrt{(-1)^2 + 5^2}$$

$$= \sqrt{1+25}$$

$$= \sqrt{26}$$

$$\approx 5.1$$

$$EF = \sqrt{[-8-(-2)]^2 + (3-6)^2}$$

$$= \sqrt{(-6)^2 + (-3)^2}$$

$$= \sqrt{36+9}$$

$$= \sqrt{45}$$

$$\approx 6.7$$

$$FD = \sqrt{[-1-(-8)]^2 + (1-3)^2}$$

$$= \sqrt{7^2 + (-2)^2}$$

$$= \sqrt{49+4}$$

$$= \sqrt{53}$$

$$\approx 7.3$$

$AB = EF$ and $CA = FD$, but $BC \neq DE$; the triangles are not congruent.

Lesson 2.3: Equation of a Circle, page 91

1. a) From the graph, x-intercepts at (-7, 0) and (7, 0).

b) From the graph, y-intercepts at (0, -7) and (0, 7).

c) If the circle has a centre at (0, 0), then the distance formula shows that this circle has a radius of 7.

d) Substitute the length of the radius into the equation of the circle.

$$x^2 + y^2 = r^2$$

$$x^2 + y^2 = 7^2$$

$$x^2 + y^2 = 49$$

2. a) $x^2 + y^2 = r^2$
 $x^2 + y^2 = 3^2$
 $x^2 + y^2 = 9$
 b) $x^2 + y^2 = r^2$
 $x^2 + y^2 = 50^2$
 $x^2 + y^2 = 2500$

c) $x^2 + y^2 = r^2$
 $x^2 + y^2 = \left(2\frac{1}{3}\right)^2$
 $x^2 + y^2 = \left(\frac{7}{3}\right)^2$

$x^2 + y^2 = \frac{49}{9}$
 d) $x^2 + y^2 = r^2$
 $x^2 + y^2 = 400^2$
 $x^2 + y^2 = 160000$
 e) $x^2 + y^2 = r^2$
 $x^2 + y^2 = 0.25^2$
 $x^2 + y^2 = 0.0625$

3. a) i) In this equation, $r^2 = 36$. Taking the positive square root shows that this circle has a radius of 6: $\sqrt{36} = 6$.

ii) To determine the x-intercepts, let $y = 0$.

$$\begin{aligned}x^2 + y^2 &= 36 \\x^2 + 0^2 &= 36 \\x^2 &= 36 \\x &= -6 \text{ or } 6\end{aligned}$$

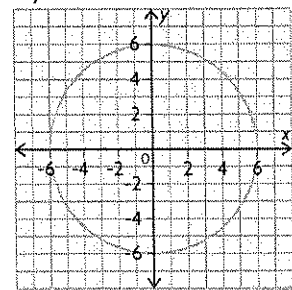
Therefore, the x-intercepts are $(-6, 0)$ and $(6, 0)$.

To determine the y-intercepts, let $x = 0$.

$$\begin{aligned}x^2 + y^2 &= 36 \\0^2 + y^2 &= 36 \\y^2 &= 36 \\y &= -6 \text{ or } 6\end{aligned}$$

Therefore, the y-intercepts are $(0, -6)$ and $(0, 6)$.

iii)



b) i) In this equation, $r^2 = 49$. Taking the positive square root shows that this circle has a radius of 7: $\sqrt{49} = 7$.

ii) To determine the x-intercepts, let $y = 0$.

$$\begin{aligned}x^2 + y^2 &= 49 \\x^2 + 0^2 &= 49 \\x^2 &= 49 \\x &= -7 \text{ or } 7\end{aligned}$$

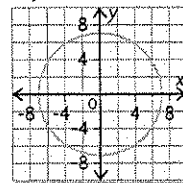
Therefore, the x-intercepts are $(-7, 0)$ and $(7, 0)$.

To determine the y-intercepts, let $x = 0$.

$$\begin{aligned}x^2 + y^2 &= 49 \\0^2 + y^2 &= 49 \\y^2 &= 49 \\y &= -7 \text{ or } 7\end{aligned}$$

Therefore, the y-intercepts are $(0, -7)$ and $(0, 7)$.

iii)



c) i) In this equation, $r^2 = 0.04$. Taking the positive square root shows that this circle has a radius of 0.2: $\sqrt{0.04} = 0.02$.

ii) To determine the x-intercepts, let $y = 0$.

$$\begin{aligned}x^2 + y^2 &= 0.04 \\x^2 + 0^2 &= 0.04 \\x^2 &= 0.04 \\x &= -0.2 \text{ or } 0.2\end{aligned}$$

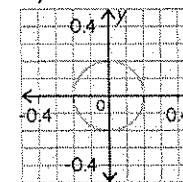
Therefore, the x-intercepts are $(-0.2, 0)$ and $(0.2, 0)$.

To determine the y-intercepts, let $x = 0$.

$$\begin{aligned}x^2 + y^2 &= 0.04 \\0^2 + y^2 &= 0.04 \\y^2 &= 0.04 \\y &= -0.2 \text{ or } 0.2\end{aligned}$$

Therefore, the y-intercepts are $(0, -0.2)$ and $(0, 0.2)$.

iii)



d) i) In this equation, $r^2 = 169$. Taking the positive square root shows this circle has a radius of 13: $\sqrt{169} = 13$.

ii) To determine the x-intercepts, let $y = 0$.

$$\begin{aligned}x^2 + y^2 &= 169 \\x^2 + 0^2 &= 169 \\x^2 &= 169 \\x &= -13 \text{ or } 13\end{aligned}$$

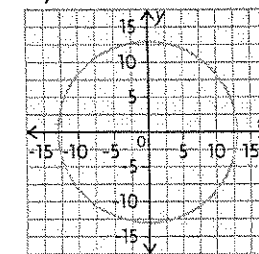
Therefore, the x-intercepts are $(-13, 0)$ and $(13, 0)$.

To determine the y-intercepts, let $x = 0$.

$$\begin{aligned}x^2 + y^2 &= 169 \\0^2 + y^2 &= 169 \\y^2 &= 169 \\y &= -13 \text{ or } 13\end{aligned}$$

Therefore, the y-intercepts are $(0, -13)$ and $(0, 13)$.

iii)



4. a) Substitute the radius of 11 into the equation for a circle.

$$\begin{aligned}x^2 + y^2 &= r^2 \\x^2 + y^2 &= 11^2 \\x^2 + y^2 &= 121\end{aligned}$$

b) Substitute one of these coordinates into the equation for a circle to determine the value of r^2 .

$$\begin{aligned}x^2 + y^2 &= r^2 \\9^2 + 0^2 &= r^2 \\r^2 &= 81\end{aligned}$$

Thus, the equation becomes $x^2 + y^2 = 81$.

c) Substitute one of these coordinates into the equation for a circle to determine the value of r^2 .

$$\begin{aligned}x^2 + y^2 &= r^2 \\0^2 + (-4)^2 &= r^2 \\r^2 &= 16\end{aligned}$$

Thus, the equation becomes $x^2 + y^2 = 16$.

d) If the circle has a diameter of 12 units, then the radius is one half of this, or 6 units. Substitute this value into the equation for a circle.

$$\begin{aligned}x^2 + y^2 &= r^2 \\x^2 + y^2 &= 6^2 \\x^2 + y^2 &= 36\end{aligned}$$

5. a) Substitute (8, 15) into the equation of a circle to solve for r .

$$\begin{aligned}x^2 + y^2 &= r^2 \\8^2 + 15^2 &= r^2 \\r^2 &= 64 + 225 \\r^2 &= 289 \\r &= 17\end{aligned}$$

Therefore this circle has a radius of 17 units.

b) Substitute the radius of 17 into the equation of a circle.

$$\begin{aligned}x^2 + y^2 &= r^2 \\x^2 + y^2 &= 17^2 \\x^2 + y^2 &= 289\end{aligned}$$

c) Solve for the x-intercepts by letting $y = 0$.

$$\begin{aligned}x^2 + y^2 &= 289 \\x^2 + 0^2 &= 289 \\x^2 &= 289 \\x &= -17 \text{ or } 17\end{aligned}$$

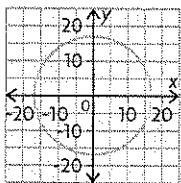
Therefore, the x-intercepts are $(-17, 0)$ and $(17, 0)$.

Solve for the y-intercepts by letting $x = 0$.

$$\begin{aligned}x^2 + y^2 &= 289 \\0^2 + y^2 &= 289 \\y^2 &= 289 \\y &= -17 \text{ or } 17\end{aligned}$$

Therefore, the y-intercepts are $(0, -17)$ and $(0, 17)$.

Use this information to sketch the graph.



6. Check whether the points satisfy the equation.

$$\begin{aligned}\text{a) } LS &= x^2 + y^2 \\&= (-4)^2 + 7^2 \\&= 16 + 49 \\&= 65\end{aligned}$$

$$RS = 65$$

$$LS = RS$$

Therefore, $(-4, 7)$ lies on the circle.

$$\begin{aligned}\text{b) } LS &= x^2 + y^2 \\&= 5^2 + (-6)^2 \\&= 25 + 36 \\&= 61\end{aligned}$$

$$RS = 65$$

$$LS \neq RS$$

Therefore, $(5, -6)$ does not lie on the circle.

$$\begin{aligned}\text{c) } LS &= x^2 + y^2 \\&= 8^2 + (-1)^2 \\&= 64 + 1 \\&= 65\end{aligned}$$

$$RS = 65$$

$$LS = RS$$

Therefore, $(8, -1)$ lies on the circle.

$$\begin{aligned}\text{d) } LS &= x^2 + y^2 \\&= (-3)^2 + (-6)^2 \\&= 9 + 36 \\&= 45\end{aligned}$$

$$RS = 65$$

$$LS \neq RS$$

Therefore, $(-3, -6)$ does not lie on the circle.

7. a) Substitute the given point into the equation of a circle to solve for r .

$$\begin{aligned}\text{i) } x^2 + y^2 &= r^2 \\(-3)^2 + 4^2 &= r^2 \\9 + 16 &= r^2 \\25 &= r^2\end{aligned}$$

Take the positive root, because a radius must be positive.

$$r = 5$$

The radius is 5 units.

$$\begin{aligned}\text{iii) } x^2 + y^2 &= r^2 \\0^2 + (-3)^2 &= r^2 \\0 + 9 &= r^2 \\9 &= r^2\end{aligned}$$

Take the positive root, because a radius must be positive.

$$r = 3$$

The radius is 3 units.

b) Substitute the value of r into the equation of a circle in each case.

$$\begin{aligned}\text{i) } x^2 + y^2 &= r^2 \\x^2 + y^2 &= 5^2 \\x^2 + y^2 &= 25\end{aligned}$$

$$\begin{aligned}\text{iii) } x^2 + y^2 &= r^2 \\x^2 + y^2 &= 3^2 \\x^2 + y^2 &= 9\end{aligned}$$

$$\begin{aligned}\text{ii) } x^2 + y^2 &= r^2 \\(5)^2 + 0^2 &= r^2 \\25 + 0 &= r^2 \\25 &= r^2\end{aligned}$$

Take the positive root, because a radius must be positive.

$$r = 5$$

The radius is 5 units.

$$\begin{aligned}\text{iv) } x^2 + y^2 &= r^2 \\8^2 + (-15)^2 &= r^2 \\64 + 225 &= r^2 \\289 &= r^2\end{aligned}$$

Take the positive root, because a radius must be positive.

$$r = 17$$

The radius is 17 units.

$$\begin{aligned}\text{ii) } x^2 + y^2 &= r^2 \\x^2 + y^2 &= 5^2 \\x^2 + y^2 &= 25\end{aligned}$$

$$\begin{aligned}\text{iv) } x^2 + y^2 &= r^2 \\x^2 + y^2 &= 17^2 \\x^2 + y^2 &= 289\end{aligned}$$

c) For example, switch the signs of the x-coordinates and/or y-coordinates of the given points to determine new points on the circle. Or, switch x-coordinates with the y-coordinates if the point lies on the x-axis or y-axis.

i) (3, 4), (-3, -4)

ii) (0, 5), (-5, 0)

iii) (3, 0), (-3, 0)

iv) (8, 15), (-8, -15)

d) First, determine the intercepts for each circle. Then, use the intercepts, the given points, and the points found in part c) to sketch the graph.

i) Let $x = 0$.

$$0^2 + y^2 = 25$$

$$y^2 = 25$$

$$y = -5 \text{ or } 5$$

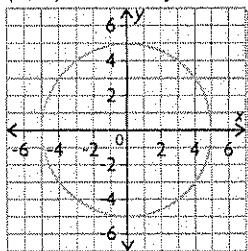
Let $y = 0$.

$$x^2 + 0^2 = 25$$

$$x^2 = 25$$

$$x = -5 \text{ or } 5$$

Therefore, the circle has x-intercepts at (-5, 0) and (5, 0), and has y-intercepts at (0, -5) and (0, 5).



ii) Let $x = 0$.

$$0^2 + y^2 = 25$$

$$y^2 = 25$$

$$y = -5 \text{ or } 5$$

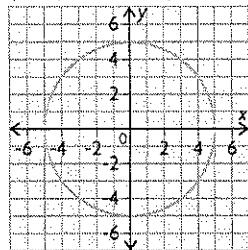
Let $y = 0$.

$$x^2 + 0^2 = 25$$

$$x^2 = 25$$

$$x = -5 \text{ or } 5$$

Therefore, the circle has x-intercepts at (-5, 0) and (5, 0), and has y-intercepts at (0, -5) and (0, 5).



iii) Let $x = 0$.

$$0^2 + y^2 = 9$$

$$y^2 = 9$$

$$y = -3 \text{ or } 3$$

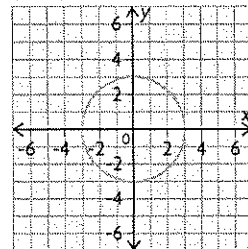
Let $y = 0$.

$$x^2 + 0^2 = 9$$

$$x^2 = 9$$

$$x = -3 \text{ or } 3$$

Therefore, the circle has x-intercepts at (-3, 0) and (3, 0), and has y-intercepts at (0, -3) and (0, 3).



iv) Let $x = 0$.

$$0^2 + y^2 = 289$$

$$y^2 = 289$$

$$y = -17 \text{ or } 17$$

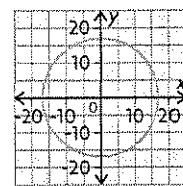
Let $y = 0$.

$$x^2 + 0^2 = 289$$

$$x^2 = 289$$

$$x = -17 \text{ or } 17$$

The circle has x-intercepts at (-17, 0) and (17, 0), and has y-intercepts at (0, -17) and (0, 17).



8. a) Let the seismograph station lie at the origin. Then, the epicentre of the earthquake is located on a circle with the equation at the right:

$$x^2 + y^2 = r^2$$

$$x^2 + y^2 = 144^2$$

$$x^2 + y^2 = 20\,736$$

b) Let the centre of the Earth lie at the origin. Then, the satellite travels along a circular orbit with the equation at the right:

$$x^2 + y^2 = r^2$$

$$x^2 + y^2 = 19\,000^2$$

$$x^2 + y^2 =$$

$$361\,000\,000$$

c) The rim is a circle of diameter 69 cm, or radius 34.5 cm. Let the centre of the rim be the origin.

$$x^2 + y^2 = r^2$$

$$x^2 + y^2 = 34.5^2$$

$$x^2 + y^2 = 1190.25$$

Then, the rim is described by a circle with the equation at the right:

d) The tunnel is a circle of diameter 2.4 m, or radius 1.2 m. Let the centre of the tunnel be the origin. Then, the walls of the tunnel are described by a circle with the following equation:

$$x^2 + y^2 = r^2$$

$$x^2 + y^2 = 1.2^2$$

$$x^2 + y^2 = 1.44$$

9. The flowerbed has diameter of 6.0 m; so, it has a radius of 3.0 m. Similarly, the garden itself has a radius of 12.0 m. Let w represent the widths of the noncircular gardens. The sum of the radius of the flowerbed, twice the width of the paths, and twice the width of the noncircular gardens, is equal to the total radius of the garden. Solve for w .

$$3 + 2(1.5) + 2w = 12$$

$$6 + 2w = 12$$

$$2w = 6$$

$$w = 3$$

Don needs to draw five circles. The first outlines the edge of the flowerbed of radius 3.0 m. Use this radius to determine the equation of the first circle.

$$x^2 + y^2 = r^2$$

$$x^2 + y^2 = 3^2$$

$$x^2 + y^2 = 9$$

The next circle outlines the inner edge of the inner series of noncircular gardens. Its radius is equal to the sum of that of the flowerbed and the width of one path.

$$r = 3 + 1.5$$

$$r = 4.5$$

$$x^2 + y^2 = r^2$$

$$x^2 + y^2 = 4.5^2$$

$$x^2 + y^2 = 20.25$$

The third circle outlines the outer edge of the inner series of noncircular gardens. Its radius is equal to the sum of that of the flowerbed, the width of one path, and the width of one noncircular garden.

$$r = 3 + 1.5 + 3$$

$$r = 7.5$$

$$x^2 + y^2 = r^2$$

$$x^2 + y^2 = 7.5^2$$

$$x^2 + y^2 = 56.25$$

The fourth circle outlines the inner edge of the outer series of noncircular gardens. Its radius is equal to the difference between the radius of the entire garden and the width of one noncircular garden.

$$r = 12 - 3$$

$$r = 9$$

$$x^2 + y^2 = r^2$$

$$x^2 + y^2 = 9^2$$

$$x^2 + y^2 = 81$$

The final circle outlines the garden itself. Use the radius of the garden to determine its equation.

$$x^2 + y^2 = r^2$$

$$x^2 + y^2 = 12^2$$

$$x^2 + y^2 = 144$$

Therefore, Don needs to draw circles with equations:

$$x^2 + y^2 = 9, x^2 + y^2 = 20.25, x^2 + y^2 = 56.25, x^2 + y^2 = 81, \text{ and } x^2 + y^2 = 144.$$

10. First, determine the radius of the orbit of the first satellite. Its equation shows that $r_1^2 = 56\,250\,000$; so, take the positive root of this number to find its orbital radius. $r_1 = 7500$. The second satellite orbits with a radius 200 km longer than the first; so, it has $r_2 = r_1 + 200$. $r_2 = 7700$. Calculate difference between the circumferences of the orbits of each satellite.

$$\begin{aligned} C_1 &= 2\pi r_1 \\ &= 2\pi(7500) \\ &\approx 47\,123.9 \end{aligned}$$

$$\begin{aligned} C_2 &= 2\pi r_2 \\ &= 2\pi(7700) \\ &\approx 48\,380.5 \end{aligned}$$

$$\begin{aligned} C_2 - C_1 &\approx 48\,380.5 - 47\,123.9 \\ &\approx 1257 \end{aligned}$$

Therefore, in one orbit, the second satellite travels about 1257 km farther than the first.

11. a) Substitute this value into the equation of the circle to solve for r^2 .

$$\begin{aligned} x^2 + y^2 &= r^2 \\ 5^2 + (-12)^2 &= r^2 \\ 25 + 144 &= r^2 \\ r^2 &= 169 \end{aligned}$$

This circle has equation $x^2 + y^2 = 169$.

b) The centre of the circle lies at the midpoint of the diameter. Therefore, the rise and run from one endpoint to the centre must be the same as the rise and run from the centre to the other endpoint.

$$\begin{aligned} \text{run} &= 0 - 5 \\ &= -5 \\ \text{rise} &= 0 - (-12) \\ &= 12 \end{aligned}$$

Thus, the other endpoint is located at $(0 - 5, 0 + 12) = (-5, 12)$.

12. Substitute the coordinates of the two endpoints into the midpoint formula to find the centre of the circle.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$\begin{aligned} C &= \left(\frac{-8 + 8}{2}, \frac{15 + (-15)}{2} \right) \\ &= (0, 0) \end{aligned}$$

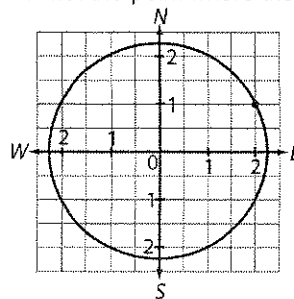
The circle has its centre at the origin; so, the standard equation applies.

Substitute one of the coordinates of the endpoints into the equation to solve for r^2 .

$$\begin{aligned} x^2 + y^2 &= r^2 \\ (-8)^2 + 15^2 &= r^2 \\ 64 + 225 &= r^2 \\ r^2 &= 289 \end{aligned}$$

Therefore, this circle has equation $x^2 + y^2 = 289$.

13. Let the point where the rock hit the water be the origin.



The boat has coordinates (2, 1). Use the equation of a circle to determine the radius of the ripple when it hits the boat.

$$\begin{aligned} x^2 + y^2 &= r^2 \\ 2^2 + 1^2 &= r^2 \\ 4 + 1 &= r^2 \\ r^2 &= 5 \end{aligned}$$

Take the positive root, since the radius is always greater than zero.

$$r \approx 2.24$$

The radius is about 2.24 m

The ripple increases in radius at a rate of 6 cm/s. If it hits the boat when $r \approx 2.24$ m, then:

$$\begin{aligned} t &\approx 224 \div 6 \\ t &\approx 37 \end{aligned}$$

Therefore, the ripple hits the boat after approximately 37 s.

14. Substitute these coordinates into the equation of the circle to solve for a and b .

$$\begin{aligned} x^2 + y^2 &= 125 \\ a^2 + 5^2 &= 125 \\ a^2 + 25 &= 125 \\ a^2 &= 100 \\ a &= -10 \text{ or } 10 \end{aligned}$$

Therefore, the first coordinate lies on the circle when $a = -10$ or 10 .

$$\begin{aligned} x^2 + y^2 &= 125 \\ 9^2 + b^2 &= 125 \\ 81 + b^2 &= 125 \\ b^2 &= 44 \\ b &\approx -6.6 \text{ or } 6.6 \end{aligned}$$

Therefore, the second coordinate lies on the circle when $b \approx -6.6$ or 6.6 .

15. For example, use the distance to determine the distance from (0, 0) to (12 504, 16 050), to find the radius of the orbit of the second satellite. The first satellite has $r^2 = 45\,000\,000$; so, take the positive root of this number to find the orbital radius of the first satellite. Compare the two radii. The second satellite ends up being outside the orbit of the first.

16. The small square has sides that are tangential to the small circle at its x-intercepts and y-intercepts.

Let $x = 0$.

$$\begin{aligned} 0^2 + y^2 &= 16 \\ y^2 &= 16 \\ y &= -4 \text{ or } 4 \end{aligned}$$

Let $y = 0$.

$$\begin{aligned} x^2 + 0^2 &= 16 \\ x^2 &= 16 \\ x &= -4 \text{ or } 4 \end{aligned}$$

Therefore, the circle has intercepts at $(-4, 0)$, $(4, 0)$, $(0, -4)$, and $(0, 4)$. Each corner of the square is in contact with the large circle; so, use the coordinates of one corner to determine its equation. Each corner of the square lies at a point horizontal to a y-intercept, and vertical to an x-intercept. Therefore, $(4, 4)$ is one of the corners of the small square.

$$\begin{aligned} x^2 + y^2 &= r^2 \\ 4^2 + 4^2 &= r^2 \\ 16 + 16 &= r^2 \\ r^2 &= 32 \end{aligned}$$

Therefore, the large circle has equation $x^2 + y^2 = 32$. Again, the large square is tangential to this circle at its intercepts.

Let $x = 0$.

$$\begin{aligned} 0^2 + y^2 &= 32 \\ y^2 &= 32 \\ y &= -5.66 \text{ or } 5.66 \end{aligned}$$

Let $y = 0$.

$$\begin{aligned} x^2 + 0^2 &= 32 \\ x^2 &= 32 \\ x &= -5.66 \text{ or } 5.66 \end{aligned}$$

The circle has intercepts at $(-5.66, 0)$, $(5.66, 0)$, $(0, -5.66)$, and $(0, 5.66)$. The side length of the large square is equal to the distance between a pair of opposite intercepts. Taking the two x-intercepts, the side length is about $5.66 - (-5.66) = 11.3$. Therefore, the large square is about 11.3 units by 11.3 units in size.

17. For example, i) The distance from the origin (0, 0) to

the point (x, y) is $\sqrt{(x-0)^2 + (y-0)^2}$, which is the same as $\sqrt{x^2 + y^2}$. But if (x, y) is a point on a circle with centre (0, 0) and radius r then the distance from (0, 0) to (x, y) is r . So $\sqrt{x^2 + y^2} = r$, and if you square both sides, $x^2 + y^2 = r^2$.

ii) The equation $x^2 + y^2 = r^2$ follows directly from the Pythagorean theorem applied to the right triangle with vertices at (0, 0), $(x, 0)$, and (x, y) .

iii) Using the formula $x^2 + y^2 = r^2$, we see that the x- and y-intercepts are $(r, 0)$, $(-r, 0)$, $(0, r)$, $(0, -r)$. This makes sense for a circle as all intercepts should be the same distance from the centre.

18. a) Recall from Chapter 1 that multiplying or dividing an equation by a non-zero constant does not change its graph. So, divide this equation by 9 to put it in standard form.

$$\begin{aligned} 9x^2 + 9y^2 &= 16 & r^2 &= \frac{16}{9}, \text{ so } r = \frac{4}{3} \\ \frac{9x^2 + 9y^2}{9} &= \frac{16}{9} \\ x^2 + y^2 &= \frac{16}{9} \end{aligned}$$

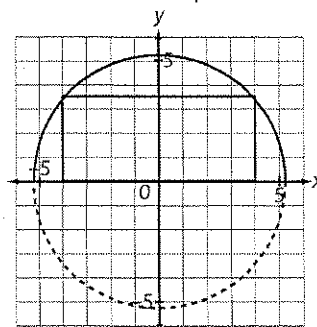
Thus, this is a circle with a centre at (0, 0) and a radius of $r = \frac{4}{3}$.

b) This is the equation of a circle that does not have its centre at the origin. Imagine translating the coordinate system such that this equation looks like an equation for a circle in standard form. Let the coordinates of this new system be x' and y' . Then, $x' = x - 2$, and $y' = y + 4$, so that the equation becomes $x'^2 + y'^2 = 9$.

In this equation, $r^2 = 9$; so, $r = 3$. Then this equation is the equation of a circle of radius 3, centred at $x' = 0$, $y' = 0$. But, $x' = x - 2$, so $x' = 0$ when $x = 2$. Similarly, $y' = y + 4$, so $y' = 0$ when $y = -4$. Thus, in the standard coordinate system, this is a circle of radius 3 centred at $(2, -4)$.

This can also be viewed in terms of transformations. The $x - 2$ term means that the graph is shifted right 2. The $y + 4$ term means that the graph is shifted down 4. Once again, this produces a circle of radius 3, centred at $(2, -4)$.

19. Sketch a diagram of the situation. Let the origin be located at the midpoint of the road.



The maximum height of the tunnel is 5.25 m; so, it is a semicircle with $r = 5.25$ m. Whether or not the truck fits depends on whether it can fit the top corners of its load through.

Determine the distance from the origin to one corner using the distance formula. Note that each corner is located half of the load-width away from the origin, or 4 m.

$$\begin{aligned}
 d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(4 - 0)^2 + (3.5 - 0)^2} \\
 &= \sqrt{16 + 12.25} \\
 &= \sqrt{28.25} \\
 &\approx 5.32
 \end{aligned}$$

Therefore, the truck requires a radius of about 5.32 m to fit through, but the tunnel only has a radius of 5.25 m. The truck will not fit.

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$$\begin{aligned}
 1. a) M &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\
 &= \left(\frac{-1 + (-7)}{2}, \frac{-2 + 10}{2} \right) \\
 &= (-4, 4)
 \end{aligned}$$

$$\begin{aligned}
 b) M &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\
 &= \left(\frac{5 + (-2)}{2}, \frac{-1 + 9}{2} \right) \\
 &= \left(\frac{3}{2}, 4 \right) \\
 &= (1.5, 4)
 \end{aligned}$$

$$\begin{aligned}
 c) M &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\
 &= \left(\frac{0 + 0}{2}, \frac{-4 + 12}{2} \right) \\
 &= (0, 4)
 \end{aligned}$$

$$\begin{aligned}
 d) M &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\
 &= \left(\frac{6 + 0}{2}, \frac{4 + 0}{2} \right) \\
 &= (3, 2)
 \end{aligned}$$

2. The centre of the circle occurs at the midpoint between the endpoints of the diameters.

$$\begin{aligned}
 M &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\
 &= \left(\frac{9 + 3}{2}, \frac{-4 + (-2)}{2} \right) \\
 &= (6, -3)
 \end{aligned}$$

Therefore, the centre of the circle exists at (6, -3).

3. All points that are the same distance from both A and B lie on the perpendicular bisector to the segment connecting them.

First, determine the midpoint of segment AB.

$$\begin{aligned}
 M &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\
 M_{AB} &= \left(\frac{-3 + 5}{2}, \frac{-1 + 3}{2} \right) \\
 &= (1, 1)
 \end{aligned}$$

The slope of the perpendicular bisector will be equal to the negative reciprocal of this slope, or -2.

Next, determine the slope of the segment.

$$\begin{aligned}
 m &= \frac{y_2 - y_1}{x_2 - x_1} \\
 m_{AB} &= \frac{3 - (-1)}{5 - (-3)} \\
 &= 0.5
 \end{aligned}$$

Substitute the midpoint into the equation of the perpendicular bisector so far to solve for its y-intercept.

$$\begin{aligned}
 y &= mx + b \\
 1 &= -2(1) + b \\
 b &= 3
 \end{aligned}$$

Therefore, points that are the same distance from both A and B lie on the perpendicular bisector of equation $y = -2x + 3$.

4. All points that are the same distance from both towns lie on the perpendicular bisector to a line segment connecting them.

First, determine the midpoint of this segment.

$$\begin{aligned}
 M &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\
 &= \left(\frac{1 + 8}{2}, \frac{7 + 5}{2} \right) \\
 &= \left(\frac{9}{2}, 6 \right)
 \end{aligned}$$

Next, determine the slope of the segment.

$$\begin{aligned}
 m &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{5 - 7}{8 - 1} \\
 &= -\frac{2}{7}
 \end{aligned}$$

The slope of the perpendicular bisector will be equal to the negative reciprocal of this slope or $\frac{7}{2}$.

Substitute the midpoint into the equation of the perpendicular bisector so far to solve for its y-intercept.

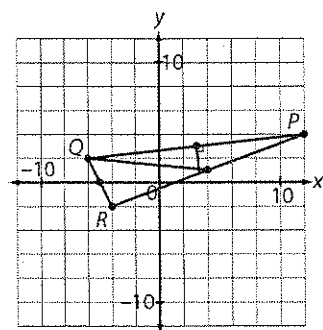
$$\begin{aligned}
 y &= mx + b \\
 6 &= \frac{7}{2} \left(\frac{9}{2} \right) + b \\
 \frac{24}{4} &= \frac{63}{4} + b \\
 b &= -\frac{39}{4}
 \end{aligned}$$

The arena can lie anywhere along the line $y = \frac{7}{2}x - \frac{39}{4}$.

5. a) Use $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$.

$$M_{PQ} = \left(\frac{12 + (-6)}{2}, \frac{4 + 2}{2} \right) = (3, 3) \quad M_{QR} = \left(\frac{-6 + (-4)}{2}, \frac{2 + (-2)}{2} \right) = (-5, 0)$$

$$M_{PR} = \left(\frac{12 + (-4)}{2}, \frac{4 + (-2)}{2} \right) = (4, 1)$$



b) The median from vertex Q joins with M_{PR} . Substitute these coordinates into the formula to determine the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \\ m_{QM_{PR}} = \frac{1 - 2}{4 - (-6)} = -\frac{1}{10}$$

Substitute the coordinates of Q into the equation of the median so far to solve for the y-intercept.

$$y = m_{QM_{PR}}x + b \\ 2 = -\frac{1}{10}(-6) + b \\ \frac{20}{10} = \frac{6}{10} + b \\ b = \frac{7}{5}$$

Therefore, the equation of the median is $y = -\frac{1}{10}x + \frac{7}{5}$.

c) Determine the slope of side PQ.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \\ m_{PQ} = \frac{2 - 4}{-6 - 12} = \frac{1}{9}$$

The slope of its perpendicular bisector will be equal to the negative reciprocal of this slope, or -9 . Substitute the coordinates of the midpoint of PQ into the equation so far to solve for the y-intercept.

$$y = mx + b \\ 3 = -9(3) + b \\ 3 = -27 + b \\ b = 30$$

Therefore, the equation of the perpendicular bisector is $y = -9x + 30$.

6. Use the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

$$\begin{aligned} \text{a) } d &= \sqrt{(7 - 2)^2 + (4 - 2)^2} \\ &= \sqrt{5^2 + 2^2} \\ &= \sqrt{25 + 4} \\ &= \sqrt{29} \\ &\approx 5.4 \end{aligned}$$

The distance is about 5.4 units.

$$\begin{aligned} \text{b) } d &= \sqrt{[8 - (-3)]^2 + (-5 - 0)^2} \\ &= \sqrt{11^2 + (-5)^2} \\ &= \sqrt{121 + 25} \\ &= \sqrt{146} \\ &\approx 12.1 \end{aligned}$$

The distance is about 12.1 units.

$$\begin{aligned} \text{c) } d &= \sqrt{(-5 - 2)^2 + (9 - 9)^2} \\ &= \sqrt{(-7)^2 + 0^2} \\ &= \sqrt{49} \\ &= 7 \end{aligned}$$

The distance is 7 units.

$$\begin{aligned} \text{d) } d &= \sqrt{(12 - 9)^2 + [-4 - (-3)]^2} \\ &= \sqrt{3^2 + (-1)^2} \\ &= \sqrt{9 + 1} \\ &= \sqrt{10} \\ &\approx 3.2 \end{aligned}$$

The distance is about 3.2 units.

7. Use $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to find the length.

$$\begin{aligned} AB &= \sqrt{[7 - (-22)]^2 + (33 - 15)^2} \\ &= \sqrt{29^2 + 18^2} \\ &= \sqrt{841 + 324} \\ &= \sqrt{1165} \\ &\approx 34.13 \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(10 - 7)^2 + (18 - 33)^2} \\ &= \sqrt{3^2 + (-15)^2} \\ &= \sqrt{9 + 225} \\ &= \sqrt{234} \\ &\approx 15.30 \end{aligned}$$

$$\begin{aligned} CD &= \sqrt{(-1 - 10)^2 + (4 - 18)^2} \\ &= \sqrt{(-11)^2 + (-14)^2} \\ &= \sqrt{121 + 196} \\ &= \sqrt{317} \\ &\approx 17.80 \end{aligned}$$

Sum the lengths of each segment to determine the total length of the power line.

$$\begin{aligned}\text{Length} &= AB + BC + CD \\ &\approx 34.13 + 15.30 + 17.80 \\ &\approx 67.2\end{aligned}$$

Therefore, the power line will be about 67.2 m long.

8. First, determine the point on the line that is closest to the given point. This occurs along a line that is perpendicular to the given line, and that passes through the required point. The perpendicular line has a slope that is the negative reciprocal of the slope given by the

equation, or $-\frac{1}{3}$.

Substitute the coordinates of the given point into the equation so far to solve for the y-intercept.

$$y = mx + b$$

$$4 = -\frac{1}{3}(-4) + b$$

$$\frac{12}{3} = \frac{4}{3} + b$$

$$b = \frac{8}{3}$$

Therefore, the distance is measured along a line with

equation $y = -\frac{1}{3}x + \frac{8}{3}$. Set this equation equal to the

given equation in to determine the point of intersection, and thus the point on the line closest to the given point.

$$-\frac{1}{3}x + \frac{8}{3} = 3x - 4$$

$$-x + 8 = 9x - 12$$

$$10x = 20$$

$$x = 2$$

Substitute this value into the given equation to solve for y.

$$y = 3x - 4$$

$$y = 3(2) - 4$$

$$y = 2$$

Thus, (2, 2) is the point on the line closest to (-4, 4). Determine the distance between these two points.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(-4 - 2)^2 + (4 - 2)^2}$$

$$= \sqrt{(-6)^2 + 2^2}$$

$$= \sqrt{40}$$

$$\approx 6.3$$

The point and the line are about 6.3 units apart.

9. From the graph, the coordinates of the three points are: A(-2, 6), B(-5, 1), and C(3, -1). Determine the lengths of each segment using the formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AB = \sqrt{[-5 - (-2)]^2 + (1 - 6)^2}$$

$$= \sqrt{(-3)^2 + (-5)^2}$$

$$= \sqrt{34}$$

$$\approx 5.8$$

$$BC = \sqrt{[3 - (-5)]^2 + (-1 - 1)^2}$$

$$= \sqrt{8^2 + (-2)^2}$$

$$= \sqrt{68}$$

$$\approx 8.2$$

$$AC = \sqrt{[3 - (-2)]^2 + (-1 - 6)^2}$$

$$= \sqrt{5^2 + (-7)^2}$$

$$= \sqrt{74}$$

$$\approx 8.6$$

$AB \neq BC \neq AC$. So, the triangle has three unequal sides.

10. a) i) This is the standard equation for a circle; so, its centre is at (0, 0).

ii) The equation shows that $r^2 = 169$. Taking the positive square root reveals that this circle has a radius of 13.

Let $x = 0$.

$$0^2 + y^2 = 169$$

$$y^2 = 169$$

$$y = -13 \text{ or } 13$$

Let $y = 0$.

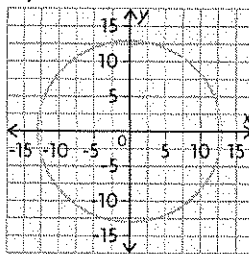
$$x^2 + 0^2 = 169$$

$$x^2 = 169$$

$$x = -13 \text{ or } 13$$

Therefore, this circle has x-intercepts at (-13, 0) and (13, 0), and y-intercepts at (0, -13) and (0, 13).

iii)



b) i) This is the standard equation for a circle; so, its centre is at (0, 0).

ii) The equation shows that $r^2 = 2.89$. Taking the positive square root reveals that this circle has a radius of 1.7.

Let $x = 0$.

$$0^2 + y^2 = 2.89$$

$$y^2 = 2.89$$

$$y = -1.7 \text{ or } 1.7$$

Let $y = 0$.

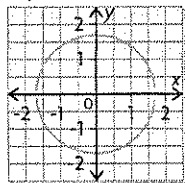
$$x^2 + 0^2 = 2.89$$

$$x^2 = 2.89$$

$$x = -1.7 \text{ or } 1.7$$

Therefore, this circle has x-intercepts at (-1.7, 0) and (1.7, 0), and y-intercepts at (0, -1.7) and (0, 1.7).

iii)



c) i) This is the standard equation for a circle; so, its centre is at (0, 0).

ii) The equation shows that $r^2 = 98$. Taking the positive square root reveals that this circle has a radius of about 9.9.

Let $x = 0$.

$$0^2 + y^2 = 98$$

$$y^2 = 98$$

$$y = -9.9 \text{ or } 9.9$$

Let $y = 0$.

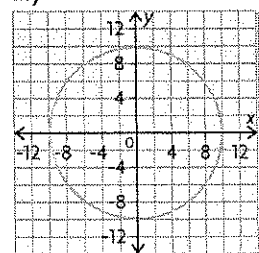
$$x^2 + 0^2 = 98$$

$$x^2 = 98$$

$$x = -9.9 \text{ or } 9.9$$

Therefore, this circle has x-intercepts at about (-9.9, 0) and (9.9, 0), and y-intercepts at about (0, -9.9) and (0, 9.9).

iii)



11. Substitute each value into the equation for a circle to solve for r^2 .

a) $x^2 + y^2 = r^2$
 $(-5)^2 + 0^2 = r^2$
 $r^2 = 25$

This circle has equation $x^2 + y^2 = 25$.

b) $x^2 + y^2 = r^2$
 $0^2 + 7^2 = r^2$
 $r^2 = 49$

This circle has equation $x^2 + y^2 = 49$.

c) $x^2 + y^2 = r^2$
 $(-3)^2 + (-8)^2 = r^2$
 $r^2 = 73$

This circle has equation $x^2 + y^2 = 73$.

d) $x^2 + y^2 = r^2$
 $4^2 + (-9)^2 = r^2$
 $r^2 = 97$

This circle has equation $x^2 + y^2 = 97$.

12. If the radius of the ripple grows at a rate of 5 cm/s, then after 6 s the ripple has a radius of $6 \times 5 = 30$ cm.

$$x^2 + y^2 = r^2$$

$$x^2 + y^2 = 30^2$$

$$x^2 + y^2 = 900$$

Therefore, the ripple has equation $x^2 + y^2 = 900$, 6 s after the raindrop hits the puddle.

13. Each point lies on a circle with $r^2 = x^2 + y^2$. Substitute each point into this equation to determine r^2 . For the given circle, $r_0^2 = 45$. The point will be inside, on, or outside the circle depending on whether r^2 is less than, equal to, or greater than this value respectively.

a) $r^2 = x^2 + y^2$
 $r^2 = 6^2 + (-3)^2$
 $r^2 = 45$
 $r^2 = r_0^2$

This point lies on the circle.

b) $r^2 = x^2 + y^2$
 $r^2 = (-1)^2 + 7^2$
 $r^2 = 50$
 $r^2 > r_0^2$

This point outside the circle.

c) $r^2 = x^2 + y^2$
 $r^2 = (-3)^2 + 5^2$
 $r^2 = 34$
 $r^2 < r_0^2$

This point lies inside the circle.

d) $r^2 = x^2 + y^2$
 $r^2 = (-7)^2 + (-2)^2$
 $r^2 = 53$
 $r^2 > r_0^2$

This point lies outside the circle.

14. a) Check whether each of these points satisfies the equation of the circle.

A(6, -7):

$$\begin{aligned} \text{LS} &= x^2 + y^2 \\ &= 6^2 + (-7)^2 \\ &= 36 + 49 \\ &= 85 \end{aligned}$$

$$\text{RS} = 85$$

$$\text{LS} = \text{RS}$$

A lies on the circle.

b) First, bisect AB by finding its midpoint.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$M_{AB} = \left(\frac{6 + 2}{2}, \frac{-7 + 9}{2} \right)$$

$$= (4, 1)$$

B(2, 9):

$$\begin{aligned} \text{LS} &= x^2 + y^2 \\ &= 2^2 + 9^2 \\ &= 4 + 81 \\ &= 85 \end{aligned}$$

$$\text{RS} = 85$$

$$\text{LS} = \text{RS}$$

B also lies on the circle.

Next, determine the slope of segment AB.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_{AB} = \frac{9 - (-7)}{2 - 6}$$

$$= -4$$

The perpendicular bisector of AB has a slope that is the negative reciprocal of

this slope, or $\frac{1}{4}$

Substitute the coordinates of the midpoint of AB into the equation so far to determine the y-intercept.

$$1 = \frac{1}{4}(4) + b$$

$$1 = 1 + b$$

$$b = 0$$

The perpendicular bisector of AB has equation $y = \frac{1}{4}x$.

c) This equation shows that the y-intercept is 0.

Therefore, this equation passes through (0, 0). (0, 0) is also the centre of the circle since this is the standard equation for a circle.

Lesson 2.4, Classifying Figures on a Coordinate Grid, page 101

1. Determine the slopes of each line segment.

$$\begin{aligned} m_{PQ} &= \frac{y_2 - y_1}{x_2 - x_1} & m_{RS} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{5 - 4}{5 - 1} & &= \frac{-3 - (-4)}{7 - 3} \\ &= \frac{1}{4} & &= \frac{1}{4} \end{aligned}$$

The line segments have the same slope. Therefore, they are parallel.

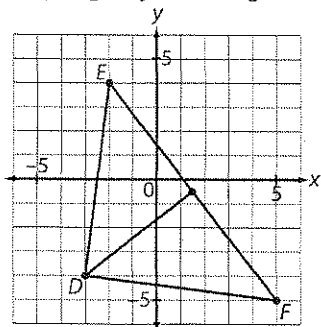
2. Determine the slopes of each line segment.

$$\begin{aligned} m_{TU} &= \frac{y_2 - y_1}{x_2 - x_1} & m_{VW} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{5 - 7}{3 - (-1)} & &= \frac{7 - 1}{-1 - (-4)} \\ &= -\frac{1}{2} & &= 2 \end{aligned}$$

The line segments have slopes that are negative reciprocals of one another. Therefore, they are perpendicular.

3. Opposite sides have the same slope; therefore, they are parallel. $ABCD$ must be a type of parallelogram. However, the slopes of adjacent sides are not negative reciprocals of one another; therefore, they are not perpendicular. $ABCD$ cannot be a square or a rectangle. Thus, $ABCD$ could be either a rhombus, or just a normal parallelogram. Information about the side lengths is required to know for certain.

4. a) Begin by sketching the triangle.



It looks like sides DE and DF are equal in length, but EF is not. Use the formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to check.

$$\begin{aligned} DE &= \sqrt{[-2 - (-3)]^2 + [4 - (-4)]^2} \\ &= \sqrt{1 + 64} \\ &= \sqrt{65} \\ &\approx 8.06 \end{aligned}$$

$$\begin{aligned} DF &= \sqrt{[5 - (-3)]^2 + [-5 - (-4)]^2} \\ &= \sqrt{64 + 1} \\ &= \sqrt{65} \\ &\approx 8.06 \end{aligned}$$

$$\begin{aligned} EF &= \sqrt{[5 - (-2)]^2 + (-5 - 4)^2} \\ &= \sqrt{49 + 81} \\ &= \sqrt{130} \\ &\approx 11.40 \end{aligned}$$

$DE = DF \neq EF$. Therefore, the triangle is isosceles.

b) First, determine the midpoint of EF .

$$\begin{aligned} M &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ M_{EF} &= \left(\frac{-2 + 5}{2}, \frac{4 + (-5)}{2} \right) \\ &= \left(\frac{3}{2}, -\frac{1}{2} \right) \end{aligned}$$

The median joins this point to vertex D . Use the formula

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to determine the length of the median.

$$\begin{aligned} DM_{EF} &= \sqrt{\left(\frac{3}{2} - (-3) \right)^2 + \left(-\frac{1}{2} - (-4) \right)^2} \\ &= \sqrt{\left(\frac{9}{2} \right)^2 + \left(\frac{7}{2} \right)^2} \\ &= \sqrt{\frac{81}{4} + \frac{49}{4}} \\ &= \sqrt{\frac{130}{4}} \\ &= \sqrt{32.5} \\ &\approx 5.70 \end{aligned}$$

The length of the median is about 5.70 units.

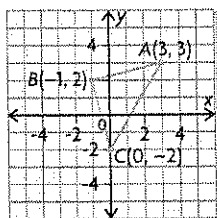
c) Determine the slopes of both the median and EF .

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} & m_{DM_{EF}} &= \frac{-\frac{1}{2} - (-4)}{\frac{3}{2} - (-3)} \\ m_{EF} &= \frac{-5 - 4}{5 - (-2)} & &= \frac{\frac{7}{2}}{\frac{9}{2}} \\ &= -\frac{9}{7} & &= \frac{7}{9} \end{aligned}$$

These slopes are negative reciprocals of one another. Therefore, the median is perpendicular to EF .

5. The fact that the side lengths of the quadrilateral are equal guarantees that it is a parallelogram and a rhombus, and possibly a square. Information about the interior angles or the slopes of the sides is required to determine whether adjacent sides are perpendicular, and thus whether it truly is a square.

6. a) For example, envisioning the coordinate grid, predict that this triangle is isosceles. Draw the triangle.



Calculate each length using $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

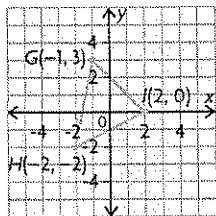
$$\begin{aligned} AB &= \sqrt{(-1 - 3)^2 + (2 - 3)^2} \\ &= \sqrt{16 + 1} \\ &= \sqrt{17} \\ &\approx 4.12 \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{[0 - (-1)]^2 + (-2 - 2)^2} \\ &= \sqrt{1 + 16} \\ &= \sqrt{17} \\ &\approx 4.12 \end{aligned}$$

$$\begin{aligned} AC &= \sqrt{(0 - 3)^2 + (-2 - 3)^2} \\ &= \sqrt{9 + 25} \\ &= \sqrt{34} \\ &\approx 5.83 \end{aligned}$$

$AB = BC \neq AC$. Therefore, the triangle is isosceles.

b) For example, envisioning the coordinate grid, predict that this triangle is scalene. Draw the triangle.



Calculate each length using $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

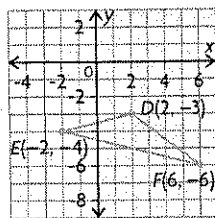
$$\begin{aligned} GH &= \sqrt{[-2 - (-1)]^2 + (-2 - 3)^2} \\ &= \sqrt{1 + 25} \\ &= \sqrt{26} \\ &\approx 5.10 \end{aligned}$$

$$\begin{aligned} HI &= \sqrt{[2 - (-2)]^2 + [0 - (-2)]^2} \\ &= \sqrt{16 + 4} \\ &= \sqrt{20} \\ &\approx 4.47 \end{aligned}$$

$$\begin{aligned} GI &= \sqrt{[2 - (-1)]^2 + (0 - 3)^2} \\ &= \sqrt{9 + 9} \\ &= \sqrt{18} \\ &\approx 4.24 \end{aligned}$$

$GH \neq HI \neq GI$. Therefore, the triangle is scalene.

c) For example, envisioning the coordinate grid, predict that this triangle is scalene. Draw the triangle.



Calculate each length using $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

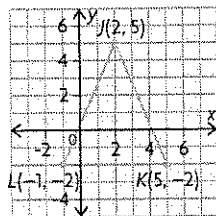
$$\begin{aligned} DE &= \sqrt{(-2 - 2)^2 + [-4 - (-3)]^2} \\ &= \sqrt{16 + 1} \\ &= \sqrt{17} \\ &\approx 4.12 \end{aligned}$$

$$\begin{aligned} EF &= \sqrt{[6 - (-2)]^2 + [-6 - (-4)]^2} \\ &= \sqrt{64 + 4} \\ &= \sqrt{68} \\ &\approx 8.25 \end{aligned}$$

$$\begin{aligned} DF &= \sqrt{(6 - 2)^2 + [-6 - (-3)]^2} \\ &= \sqrt{16 + 9} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

$DE \neq EF \neq DF$. Therefore, the triangle is scalene.

d) For example, envisioning the coordinate grid, predict that this triangle is isosceles. Draw the triangle.



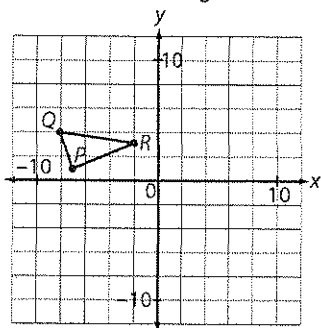
Calculate each length using $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

$$\begin{aligned} JK &= \sqrt{(5 - 2)^2 + (-2 - 5)^2} & KL &= \sqrt{(-1 - 5)^2 + [-2 - (-2)]^2} \\ &= \sqrt{9 + 49} & &= \sqrt{36 + 0} \\ &= \sqrt{58} & &= \sqrt{36} \\ &\approx 7.62 & &= 6 \end{aligned}$$

$$\begin{aligned} JL &= \sqrt{(-1 - 2)^2 + (-2 - 5)^2} \\ &= \sqrt{9 + 49} \\ &= \sqrt{58} \\ &\approx 7.62 \end{aligned}$$

$JK = JL \neq KL$. Therefore, the triangle is isosceles.

7. Sketch the triangle.



It looks as though PQ and PR may form a right angle. Determine their slopes.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_{PQ} = \frac{4 - 1}{-8 - (-7)}$$

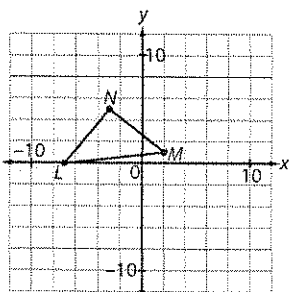
$$= -3$$

$$m_{PR} = \frac{3 - 1}{-1 - (-7)}$$

$$= \frac{1}{3}$$

These slopes are negative reciprocals of one another. Therefore, triangle PQR is a right triangle.

8. Sketch the triangle on a grid.



It appears LN and MN may form a right angle, and be the same length. Determine their slopes and lengths.

$$m_{LN} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 0}{-3 - (-7)} = \frac{5}{4}$$

$$m_{MN} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 1}{-3 - 2} = -\frac{4}{5}$$

$$LN = \sqrt{[-3 - (-7)]^2 + (5 - 0)^2} = \sqrt{16 + 25} = \sqrt{41} \approx 6.4$$

$$MN = \sqrt{(-3 - 2)^2 + (5 - 1)^2} = \sqrt{25 + 16} = \sqrt{41} \approx 6.4$$

Their slopes are negative reciprocals of one another; so, they form a right angle. Also, LN and MN are the same length. Therefore, this is a right isosceles triangle.

9. a) Use the vertex formula to determine the lengths of each side of the triangle. Check whether these lengths satisfy the relation for right-triangles given by the Pythagorean theorem, i.e. whether $a^2 + b^2 = c^2$, where c is the length of the longest side. If this equation is satisfied, then it is a right triangle; otherwise, it is not. In this particular case:

$$PQ = \sqrt{[4 - (-2)]^2 + [1 - (-3)]^2}$$

$$= \sqrt{36 + 16}$$

$$= \sqrt{52}$$

$$\approx 7.21$$

$$QR = \sqrt{(2 - 4)^2 + (4 - 1)^2}$$

$$= \sqrt{4 + 9}$$

$$= \sqrt{13}$$

$$\approx 3.61$$

$$PR = \sqrt{[2 - (-2)]^2 + [4 - (-3)]^2}$$

$$= \sqrt{16 + 49}$$

$$= \sqrt{65}$$

$$\approx 8.06$$

PR is the longest side; so, if this is right triangle, then PR will be the hypotenuse. Check whether $PR^2 = PQ^2 + QR^2$.

$$LS = PQ^2 + QR^2$$

$$\approx 7.21^2 + 3.61^2$$

$$\approx 65$$

$$RS = SU^2$$

$$\approx 8.06^2$$

$$\approx 65$$

$$LS = RS$$

Therefore, the relation for right-triangles holds true. $\triangle PQR$ is a right triangle.

b) i) Use $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to determine the lengths of each side.

$$ST = \sqrt{[-1 - (-2)]^2 + (-2 - 2)^2} = \sqrt{1 + 16} = \sqrt{17} \approx 4.12$$

$$TU = \sqrt{[7 - (-1)]^2 + [0 - (-2)]^2} = \sqrt{64 + 4} = \sqrt{68} \approx 8.25$$

$$SU = \sqrt{[7 - (-2)]^2 + (0 - 2)^2}$$

$$= \sqrt{81 + 4}$$

$$= \sqrt{85}$$

$$\approx 9.21$$

SU is the longest side; so, if this is a right triangle, then SU will be the hypotenuse. Check whether $ST^2 + TU^2 = SU^2$.

$$LS = ST^2 + TU^2$$

$$\approx 4.12^2 + 8.25^2$$

$$\approx 85$$

$$RS = SU^2$$

$$\approx 9.21^2$$

$$\approx 85$$

$$LS = RS$$

Therefore, the relation for right-triangles holds true. $\triangle STU$ is a right triangle.

ii) Use $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to determine the lengths of each side.

$$XY = \sqrt{(1 - 3)^2 + (-2 - 2)^2} = \sqrt{4 + 16} = \sqrt{20} \approx 4.47$$

$$YZ = \sqrt{(-3 - 1)^2 + [6 - (-2)]^2} = \sqrt{16 + 64} = \sqrt{80} \approx 8.94$$

$$\begin{aligned}
 XZ &= \sqrt{(-3-3)^2 + (6-2)^2} \\
 &= \sqrt{36+16} \\
 &= \sqrt{52} \\
 &\approx 7.21
 \end{aligned}$$

YZ is the longest side; so, if this is a right triangle, then YZ will be the hypotenuse.

Check whether

$$XY^2 + XZ^2 = YZ^2$$

$$\begin{aligned}
 LS &= XY^2 + XZ^2 \\
 &= 4.47^2 + 7.21^2 \\
 &= 72
 \end{aligned}$$

$$\begin{aligned}
 RS &= YZ^2 \\
 &= (8.94)^2 \\
 &= 80
 \end{aligned}$$

$$LS \neq RS$$

Therefore, the relation for right-triangles does not hold true. $\triangle XYZ$ is a not right triangle.

iii) Use $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to determine the length of each side.

$$\begin{aligned}
 AB &= \sqrt{(3-5)^2 + (8-5)^2} & BC &= \sqrt{(3-8)^2 + (8-7)^2} \\
 &= \sqrt{4+9} & &= \sqrt{25+1} \\
 &= \sqrt{13} & &= \sqrt{26} \\
 &\approx 3.61 & &\approx 5.10
 \end{aligned}$$

$$\begin{aligned}
 AC &= \sqrt{(8-5)^2 + (7-5)^2} \\
 &= \sqrt{9+4} \\
 &= \sqrt{13} \\
 &\approx 3.61
 \end{aligned}$$

BC is the longest side; so, if this is a right triangle, then BC will be the hypotenuse.

Check whether $AB^2 + AC^2 = BC^2$.

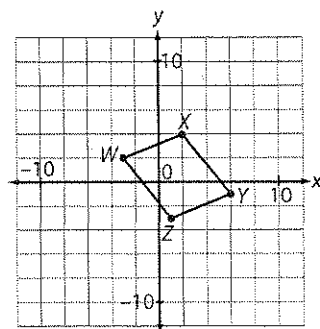
$$\begin{aligned}
 LS &= AB^2 + AC^2 \\
 &= 3.61^2 + 3.61^2 \\
 &= 26
 \end{aligned}$$

$$\begin{aligned}
 RS &= BC^2 \\
 &= 5.10^2 \\
 &= 26
 \end{aligned}$$

$$LS = RS$$

Therefore, the relation for right-triangles holds true. Also, $AB = AC$, so the triangle is isosceles. $\triangle ABC$ is a right isosceles triangle.

10. a) First, sketch the quadrilateral.



Determine the length of each side using the formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \text{ and the slope of each side}$$

$$\text{using the formula } m = \frac{y_2 - y_1}{x_2 - x_1}$$

For WX:

$$\begin{aligned}
 WX &= \sqrt{[2 - (-3)]^2 + (4 - 2)^2} & m_{WX} &= \frac{4 - 2}{2 - (-3)} \\
 &= \sqrt{25 + 4} & &= \frac{2}{5} \\
 &= \sqrt{29} & &= 0.4 \\
 &\approx 5.39
 \end{aligned}$$

For XY:

$$\begin{aligned}
 XY &= \sqrt{(6 - 2)^2 + (-1 - 4)^2} & m_{XY} &= \frac{-1 - 4}{6 - 2} \\
 &= \sqrt{16 + 25} & &= -\frac{5}{4} \\
 &= \sqrt{41} & &= -1.25 \\
 &\approx 6.40
 \end{aligned}$$

For YZ:

$$\begin{aligned}
 YZ &= \sqrt{(1 - 6)^2 + [-3 - (-1)]^2} & m_{YZ} &= \frac{-3 - (-1)}{1 - 6} \\
 &= \sqrt{25 + 4} & &= \frac{2}{5} \\
 &= \sqrt{29} & &= 0.4 \\
 &\approx 5.39
 \end{aligned}$$

For ZW:

$$\begin{aligned}
 ZW &= \sqrt{(-3 - 1)^2 + [2 - (-3)]^2} & m_{ZW} &= \frac{2 - (-3)}{-3 - 1} \\
 &= \sqrt{16 + 25} & &= -\frac{5}{4} \\
 &= \sqrt{41} & &= -1.25 \\
 &\approx 6.40
 \end{aligned}$$

b) $WX = YZ$ and $XY = ZW$. Also, $m_{WX} = m_{YZ}$ and $m_{XY} = m_{ZW}$. Therefore, this quadrilateral has opposite sides that are equal and parallel; so, it is a parallelogram.

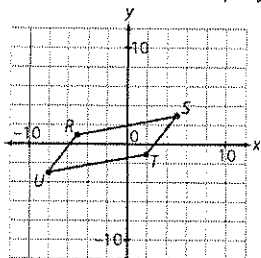
However, $\frac{2}{5}$ and $-\frac{5}{4}$ are not negative reciprocals; so, adjacent sides are not perpendicular, and the quadrilateral is not a rectangle. Also, adjacent sides are not equal in length; so, the quadrilateral is not a rhombus.

c) Determine the lengths of the diagonals, WY and XZ and calculate their difference.

$$\begin{aligned}
 WY &= \sqrt{[6 - (-3)]^2 + (-1 - 2)^2} & XZ &= \sqrt{(1 - 2)^2 + (-3 - 4)^2} \\
 &= \sqrt{81 + 9} & &= \sqrt{1 + 49} \\
 &= \sqrt{90} & &= \sqrt{50} \\
 &\approx 9.49 & &\approx 7.07 \\
 WY - XZ &\approx 9.49 - 7.07 \\
 &\approx 2.42
 \end{aligned}$$

Therefore, the difference between the lengths of the two diagonals is about 2.42 units.

11. First, sketch the polygon.



Determine the length and slope of each side using

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \text{ and } m = \frac{y_2 - y_1}{x_2 - x_1}$$

For RS:

$$\begin{aligned} RS &= \sqrt{[5 - (-5)]^2 + (3 - 1)^2} & m_{RS} &= \frac{3 - 1}{5 - (-5)} \\ &= \sqrt{100 + 4} & &= \frac{1}{5} \\ &= \sqrt{104} & & \\ &\approx 10.20 & & \end{aligned}$$

For TU:

$$\begin{aligned} TU &= \sqrt{(-8 - 2)^2 + [-3 - (-1)]^2} & m_{TU} &= \frac{-3 - (-1)}{-8 - 2} \\ &= \sqrt{100 + 4} & &= \frac{1}{5} \\ &= \sqrt{104} & & \\ &\approx 10.20 & & \end{aligned}$$

For UR:

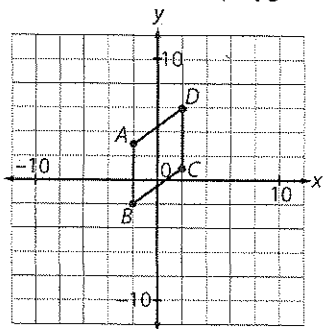
$$\begin{aligned} UR &= \sqrt{[-5 - (-8)]^2 + [1 - (-3)]^2} & m_{UR} &= \frac{1 - (-3)}{-5 - (-8)} \\ &= \sqrt{9 + 16} & &= \frac{4}{3} \\ &= \sqrt{25} & & \\ &= 5 & & \end{aligned}$$

For ST:

$$\begin{aligned} ST &= \sqrt{(2 - 5)^2 + (-1 - 3)^2} & m_{ST} &= \frac{-1 - 3}{2 - 5} \\ &= \sqrt{9 + 16} & &= \frac{4}{3} \\ &= \sqrt{25} & & \\ &= 5 & & \end{aligned}$$

$RS = TU$ and $ST = UR$. Also, $m_{RS} = m_{TU}$, and $m_{ST} = m_{UR}$. Opposite sides of this quadrilateral are equal in length, as well as parallel, since they have the same slope. This polygon is a parallelogram.

12. First, sketch the polygon.



Determine the length and slope of each side using

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \text{ and } m = \frac{y_2 - y_1}{x_2 - x_1}$$

For AB:

$$\begin{aligned} AB &= \sqrt{[-2 - (-2)]^2 + (-2 - 3)^2} & m_{AB} &= \frac{-2 - 3}{-2 - (-2)} \\ &= \sqrt{0 + 25} & &= \frac{-5}{0} \\ &= \sqrt{25} & &= \text{undefined} \\ &= 5 & & \end{aligned}$$

For BC:

$$\begin{aligned} BC &= \sqrt{[2 - (-2)]^2 + [1 - (-2)]^2} & m_{BC} &= \frac{1 - (-2)}{2 - (-2)} \\ &= \sqrt{16 + 9} & &= \frac{3}{4} \\ &= \sqrt{25} & & \\ &= 5 & & \end{aligned}$$

For CD:

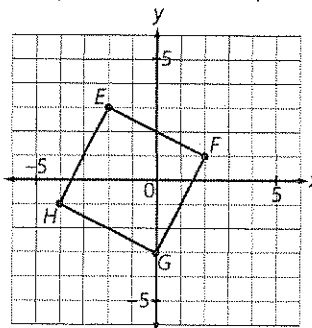
$$\begin{aligned} CD &= \sqrt{(2 - 2)^2 + (6 - 1)^2} & m_{CD} &= \frac{6 - 1}{2 - 2} \\ &= \sqrt{0 + 25} & &= \frac{5}{0} \\ &= \sqrt{25} & &= \text{undefined} \\ &= 5 & & \end{aligned}$$

For DA:

$$\begin{aligned} DA &= \sqrt{(-2 - 2)^2 + (3 - 6)^2} & m_{DA} &= \frac{3 - 6}{-2 - 2} \\ &= \sqrt{16 + 9} & &= \frac{3}{4} \\ &= \sqrt{25} & & \\ &= 5 & & \end{aligned}$$

$AB = BC = CD = DA$. Also, $m_{AB} = m_{CD}$ and $m_{BC} = m_{DA}$. All sides of the quadrilateral are equal in length, and opposite sides have the same slope; so, they are parallel. Therefore, this polygon is a rhombus.

13. a) First, sketch the quadrilateral.



Determine the length and slope of each side using

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \text{ and } m = \frac{y_2 - y_1}{x_2 - x_1}$$

For EF:

$$\begin{aligned} EF &= \sqrt{[2 - (-2)]^2 + (1 - 3)^2} & m_{EF} &= \frac{1 - 3}{2 - (-2)} \\ &= \sqrt{16 + 4} & &= \frac{-1}{2} \\ &= \sqrt{20} & & \\ &\approx 4.47 & & \end{aligned}$$

For FG :

$$\begin{aligned} FG &= \sqrt{(0-2)^2 + (-3-1)^2} \\ &= \sqrt{4+16} \\ &= \sqrt{20} \\ &\approx 4.47 \end{aligned}$$

$$\begin{aligned} m_{FG} &= \frac{-3-1}{0-2} \\ &= 2 \end{aligned}$$

For GH :

$$\begin{aligned} GH &= \sqrt{(-4-0)^2 + [-1-(-3)]^2} \\ &= \sqrt{16+4} \\ &= \sqrt{20} \\ &\approx 4.47 \end{aligned}$$

$$\begin{aligned} m_{GH} &= \frac{-1-(-3)}{-4-0} \\ &= -\frac{1}{2} \end{aligned}$$

For HE :

$$\begin{aligned} HE &= \sqrt{[-2-(-4)]^2 + [3-(-1)]^2} \\ &= \sqrt{4+16} \\ &= \sqrt{20} \\ &\approx 4.47 \end{aligned}$$

$$\begin{aligned} m_{HE} &= \frac{3-(-1)}{-2-(-4)} \\ &= 2 \end{aligned}$$

$EF = FG = GH = HE$, so all sides are equal in length. Also, $m_{EF} = m_{GH}$, and $m_{FG} = m_{HE}$; so, opposite sides are parallel, since they have the same slope. $-\frac{1}{2}$ and 2 are

negative reciprocals of one another; so, adjacent sides are perpendicular. This is a square.

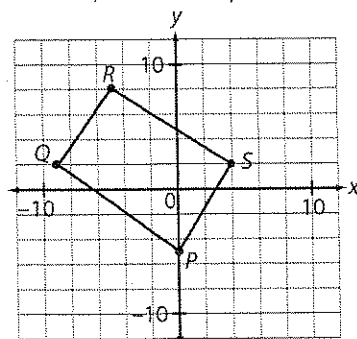
b) Determine the slopes of EG and FH , the diagonals.

$$\begin{aligned} m_{EG} &= \frac{y_2 - y_1}{x_2 - x_1} & m_{FH} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-3-3}{0-(-2)} & &= \frac{-1-1}{-4-2} \\ &= -3 & &= \frac{1}{3} \end{aligned}$$

Since $m_{EG} = -\frac{1}{m_{FH}}$, the slopes of the diagonals are

negative reciprocals of one another, and so the diagonals are perpendicular.

14. First, sketch the quadrilateral.



This quadrilateral certainly does not appear to be a rectangle. Determine the length and slope of each side

using $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ and $m = \frac{y_2 - y_1}{x_2 - x_1}$.

For PQ :

$$\begin{aligned} PQ &= \sqrt{(-9-0)^2 + [2-(-5)]^2} \\ &= \sqrt{81+49} \\ &= \sqrt{130} \\ &\approx 11.40 \end{aligned}$$

$$\begin{aligned} m_{PQ} &= \frac{2-(-5)}{-9-0} \\ &= -\frac{7}{9} \end{aligned}$$

For QR :

$$\begin{aligned} QR &= \sqrt{[-5-(-9)]^2 + (8-2)^2} \\ &= \sqrt{16+36} \\ &= \sqrt{52} \\ &\approx 7.21 \end{aligned}$$

$$\begin{aligned} m_{QR} &= \frac{8-2}{-5-(-9)} \\ &= \frac{3}{2} \end{aligned}$$

For RS :

$$\begin{aligned} RS &= \sqrt{[4-(-5)]^2 + (2-8)^2} \\ &= \sqrt{81+36} \\ &= \sqrt{117} \\ &\approx 10.82 \end{aligned}$$

$$\begin{aligned} m_{RS} &= \frac{2-8}{4-(-5)} \\ &= -\frac{2}{3} \end{aligned}$$

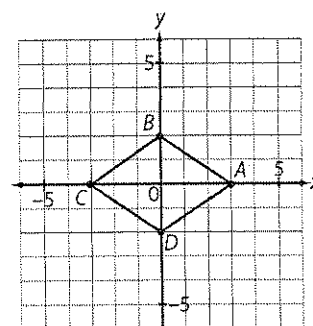
For SP :

$$\begin{aligned} SP &= \sqrt{(0-4)^2 + (-5-2)^2} \\ &= \sqrt{16+49} \\ &= \sqrt{65} \\ &\approx 8.06 \end{aligned}$$

$$\begin{aligned} m_{SP} &= \frac{-5-2}{0-4} \\ &= \frac{7}{4} \end{aligned}$$

$PQ \neq QR \neq RS \neq SP$. Opposite sides are not equal in length; therefore, the quadrilateral is not a rectangle. Also, only one pair of adjacent sides have slopes that are negative reciprocals, QR and RS . In a rectangle, all pairs of adjacent sides are perpendicular and thus have slopes that are negative reciprocals; therefore, this fact also proves $PQRS$ is not a rectangle.

15. Answers will vary, e.g., Use the distance formula to determine the lengths of the sides. If they are equal, then the quadrilateral is a rhombus. e.g., $A(3, 0)$, $B(0, 2)$, $C(-3, 0)$, $D(0, -2)$:

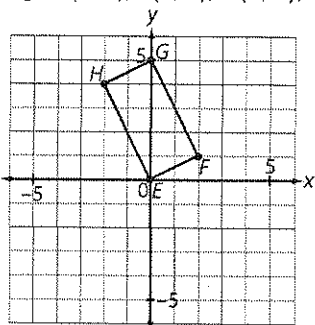


$$\begin{aligned} d_{AB} &= \sqrt{(3-0)^2 + (0-2)^2} & d_{BC} &= \sqrt{[0-(-3)]^2 + (2-0)^2} \\ &= \sqrt{3^2 + 2^2} & &= \sqrt{3^2 + 2^2} \\ &= \sqrt{9+4} & &= \sqrt{9+4} \\ &= \sqrt{13} & &= \sqrt{13} \end{aligned}$$

$$\begin{aligned} d_{CD} &= \sqrt{(-3-0)^2 + (0-2)^2} & d_{DA} &= \sqrt{[0-3]^2 + (-2-0)^2} \\ &= \sqrt{(-3)^2 + (-2)^2} & &= \sqrt{(-3)^2 + (-2)^2} \\ &= \sqrt{9+4} & &= \sqrt{9+4} \\ &= \sqrt{13} & &= \sqrt{13} \end{aligned}$$

All sides are the same length, so $ABCD$ is a rhombus. Use the slope formula to determine the slopes of the sides. If the slopes of adjacent sides are negative reciprocals of each other, then the quadrilateral is a rectangle.

e.g., $E(0, 0)$, $F(2, 1)$, $G(0, 5)$, $H(-2, 4)$.



$$m_{EF} = \frac{0-1}{0-2} = \frac{1}{2}$$

$$m_{FG} = \frac{1-5}{2-0} = -\frac{4}{2} = -2$$

$$m_{GH} = \frac{5-4}{0-(-2)} = \frac{1}{2}$$

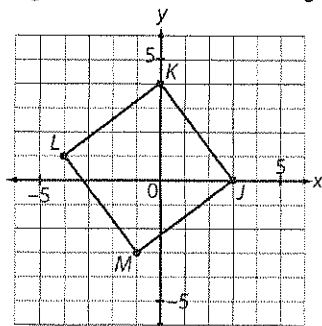
$$m_{HE} = \frac{0-4}{0-(-2)} = -\frac{4}{2} = -2$$

All sides intersect at right angles, so $EFGH$ is a rectangle.

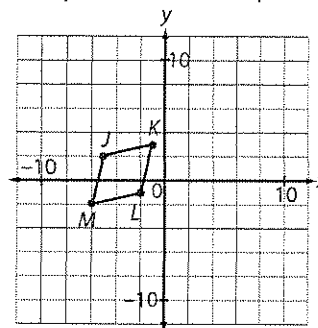
If the quadrilateral is both a rhombus and a rectangle, then it is a square.

e.g., $J(3, 0)$, $K(0, 4)$, $L(-4, 1)$, $M(-1, -3)$.

This square has opposite sides with slopes of $-\frac{4}{3}$ and $\frac{3}{4}$, and sides of length 5.



16. a) First sketch the quadrilateral.



This quadrilateral appears to be a rhombus. Determine the length and slope of each side using

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \text{ and } m = \frac{y_2 - y_1}{x_2 - x_1}$$

For JK :

$$JK = \sqrt{[-5 - (-1)]^2 + (2 - 3)^2} = \sqrt{16 + 1} = \sqrt{17} \approx 4.12$$

$$m_{JK} = \frac{2 - 3}{-5 - (-1)} = \frac{-1}{-4} = \frac{1}{4}$$

For KL :

$$KL = \sqrt{[-1 - (-2)]^2 + [3 - (-1)]^2} = \sqrt{1 + 16} = \sqrt{17} \approx 4.12$$

$$m_{KL} = \frac{3 - (-1)}{-1 - (-2)} = \frac{4}{-1} = -4$$

For LM :

$$LM = \sqrt{[-2 - (-6)]^2 + [-1 - (-2)]^2} = \sqrt{16 + 1} = \sqrt{17} \approx 4.12$$

$$m_{LM} = \frac{-1 - (-2)}{-2 - (-6)} = \frac{1}{-4} = -\frac{1}{4}$$

For JM :

$$JM = \sqrt{[-5 - (-6)]^2 + [2 - (-2)]^2} = \sqrt{1 + 16} = \sqrt{17} \approx 4.12$$

$$m_{JM} = \frac{2 - (-2)}{-5 - (-6)} = \frac{4}{-1} = -4$$

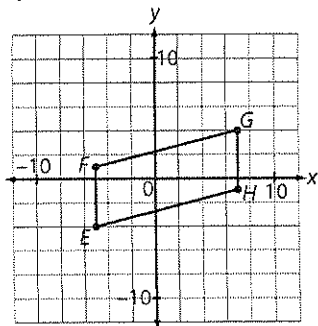
$JK = KL = LM = JM$; so, all sides are equal in length.

Also, $m_{JK} = m_{LM}$, and $m_{KL} = m_{JM}$; so, opposite sides have the same slope, and are thus parallel. The slopes of

adjacent sides, $\frac{1}{4}$ and -4 , are not negative reciprocals;

so, no adjacent sides meet at right angles. Thus, the quadrilateral is a rhombus.

b) First, sketch the quadrilateral.



This quadrilateral appears to be a parallelogram. Determine the length and slope of each side using

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \text{ and } m = \frac{y_2 - y_1}{x_2 - x_1}.$$

For EF:

$$\begin{aligned} EF &= \sqrt{[-5 - (-5)]^2 + [1 - (-4)]^2} & m_{EF} &= \frac{1 - (-4)}{-5 - (-5)} \\ &= \sqrt{0 + 25} & &= \frac{5}{0} \\ &= \sqrt{25} & &= \text{undefined} \\ &= 5 & & \end{aligned}$$

For FG:

$$\begin{aligned} FG &= \sqrt{[7 - (-5)]^2 + (4 - 1)^2} & m_{FG} &= \frac{4 - 1}{7 - (-5)} \\ &= \sqrt{144 + 9} & &= \frac{1}{4} \\ &= \sqrt{153} & & \\ &\approx 12.37 & & \end{aligned}$$

For GH:

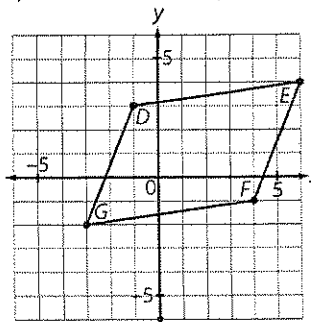
$$\begin{aligned} GH &= \sqrt{(7 - 7)^2 + (-1 - 4)^2} & m_{GH} &= \frac{-1 - 4}{7 - 7} \\ &= \sqrt{0 + 25} & &= \frac{-5}{0} \\ &= \sqrt{25} & &= \text{undefined} \\ &= 5 & & \end{aligned}$$

For HE:

$$\begin{aligned} HE &= \sqrt{(-5 - 7)^2 + [-4 - (-1)]^2} & m_{HE} &= \frac{-4 - (-1)}{-5 - 7} \\ &= \sqrt{144 + 9} & &= \frac{1}{4} \\ &= \sqrt{153} & & \\ &\approx 12.37 & & \end{aligned}$$

$EF = GH$ and $FG = HE$; so, opposite sides, but not all sides, are equal in length. Also, $m_{EF} = m_{FG}$, and $m_{FG} = m_{HE}$; so, opposite sides have the same slope, and are thus parallel. The slopes of adjacent sides, $\frac{1}{4}$ and an 'undefined' slope, are not negative reciprocals; so, no adjacent sides meet at right angles. Thus, the quadrilateral is a parallelogram.

c) First, sketch the quadrilateral.



This quadrilateral appears to be a parallelogram. Determine the length and slope of each side using

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \text{ and } m = \frac{y_2 - y_1}{x_2 - x_1}.$$

For DE:

$$\begin{aligned} DE &= \sqrt{[6 - (-1)]^2 + (4 - 3)^2} & m_{DE} &= \frac{4 - 3}{6 - (-1)} \\ &= \sqrt{49 + 1} & &= \frac{1}{7} \\ &= \sqrt{50} & & \\ &\approx 7.07 & & \end{aligned}$$

For EF:

$$\begin{aligned} EF &= \sqrt{(4 - 6)^2 + (-1 - 4)^2} & m_{EF} &= \frac{-1 - 4}{4 - 6} \\ &= \sqrt{4 + 25} & &= \frac{5}{2} \\ &= \sqrt{29} & & \\ &\approx 5.39 & & \end{aligned}$$

For FG:

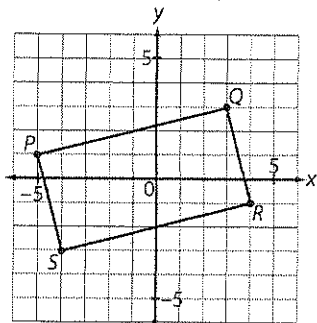
$$\begin{aligned} FG &= \sqrt{(-3 - 4)^2 + [-2 - (-1)]^2} & m_{FG} &= \frac{-2 - (-1)}{-3 - 4} \\ &= \sqrt{49 + 1} & &= \frac{1}{7} \\ &= \sqrt{50} & & \\ &\approx 7.07 & & \end{aligned}$$

For GD:

$$\begin{aligned} GD &= \sqrt{[-1 - (-3)]^2 + [3 - (-2)]^2} & m_{GD} &= \frac{3 - (-2)}{-1 - (-3)} \\ &= \sqrt{4 + 25} & &= \frac{5}{2} \\ &= \sqrt{29} & & \\ &\approx 5.39 & & \end{aligned}$$

$DE = FG$ and $EF = GD$; so, opposite sides, but not all sides, are equal in length. Also, $m_{DE} = m_{FG}$, and $m_{EF} = m_{GD}$; so, opposite sides have the same slope, and are thus parallel. The slopes of adjacent sides, $\frac{5}{2}$ and $\frac{1}{7}$, are not negative reciprocals; so, no adjacent sides meet at right angles. Thus, the quadrilateral is a parallelogram.

d) First, sketch the quadrilateral.



This quadrilateral appears to be a rectangle. Determine the length and slope of each side using

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \text{ and } m = \frac{y_2 - y_1}{x_2 - x_1}.$$

For PQ:

$$\begin{aligned} PQ &= \sqrt{[3 - (-5)]^2 + (3 - 1)^2} & m_{PQ} &= \frac{3 - 1}{3 - (-5)} \\ &= \sqrt{64 + 4} & &= \frac{2}{8} \\ &= \sqrt{68} & &= \frac{1}{4} \\ &\approx 8.25 & & \end{aligned}$$

For QR:

$$\begin{aligned} QR &= \sqrt{(4 - 3)^2 + (-1 - 3)^2} & m_{QR} &= \frac{-1 - 3}{4 - 3} \\ &= \sqrt{1 + 16} & &= -4 \\ &= \sqrt{17} & & \\ &\approx 4.12 & & \end{aligned}$$

For RS:

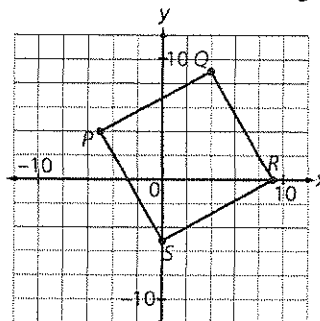
$$\begin{aligned} RS &= \sqrt{(-4 - 4)^2 + [-3 - (-1)]^2} & m_{RS} &= \frac{-3 - (-1)}{-4 - 4} \\ &= \sqrt{64 + 4} & &= \frac{-3 + 1}{-8} \\ &= \sqrt{68} & &= \frac{-2}{-8} \\ &\approx 8.25 & &= \frac{1}{4} \end{aligned}$$

For SP:

$$\begin{aligned} SP &= \sqrt{[-5 - (-4)]^2 + [1 - (-3)]^2} & m_{SP} &= \frac{1 - (-3)}{-5 - (-4)} \\ &= \sqrt{1 + 16} & &= -4 \\ &= \sqrt{17} & & \\ &\approx 4.12 & & \end{aligned}$$

$PQ = RS$ and $QR = SP$; so, opposite sides, but not all sides, are equal in length. Also, $m_{PQ} = m_{RS}$, and $m_{QR} = m_{SP}$; so, opposite sides have the same slope, and are thus parallel. Finally, the slopes of adjacent sides, $\frac{1}{4}$ and -4 , are negative reciprocals; so, adjacent sides meet at right angles. Thus, the quadrilateral is a rectangle.

17. First, sketch the building lot.



This lot appears to be a rectangle. Determine the length and slope of each side using $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$\text{and } m = \frac{y_2 - y_1}{x_2 - x_1}.$$

For AB:

$$\begin{aligned} AB &= \sqrt{[4 - (-5)]^2 + (9 - 4)^2} & m_{AB} &= \frac{9 - 4}{4 - (-5)} \\ &= \sqrt{81 + 25} & &= \frac{5}{9} \\ &= \sqrt{106} & & \\ &\approx 10.30 & & \end{aligned}$$

For BC:

$$\begin{aligned} BC &= \sqrt{(9 - 4)^2 + (0 - 9)^2} & m_{BC} &= \frac{0 - 9}{9 - 4} \\ &= \sqrt{25 + 81} & &= -\frac{9}{5} \\ &= \sqrt{106} & & \\ &\approx 10.30 & & \end{aligned}$$

For CD:

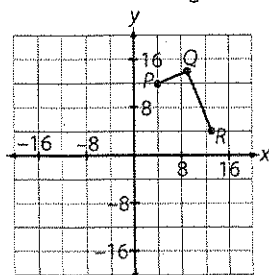
$$\begin{aligned} CD &= \sqrt{(0 - 9)^2 + (-5 - 0)^2} & m_{CD} &= \frac{-5 - 0}{0 - 9} \\ &= \sqrt{81 + 25} & &= \frac{5}{9} \\ &= \sqrt{106} & & \\ &\approx 10.30 & & \end{aligned}$$

For DA:

$$\begin{aligned} DA &= \sqrt{(-5 - 0)^2 + [4 - (-5)]^2} & m_{DA} &= \frac{4 - (-5)}{-5 - 0} \\ &= \sqrt{25 + 81} & &= -\frac{9}{5} \\ &= \sqrt{106} & & \\ &\approx 10.30 & & \end{aligned}$$

$AB = BC = CD = DA$; so, all sides are equal in length. Also, $m_{AB} = m_{CD}$, and $m_{BC} = m_{DA}$; so, opposite sides have the same slope, and are thus parallel. Finally, the slopes of adjacent sides, $\frac{5}{9}$ and $-\frac{9}{5}$, are negative reciprocals; so, adjacent sides meet at right angles. Therefore, the building lot is shaped as a square.

18. a) & b) For example, begin by sketching the known coordinates on a grid.



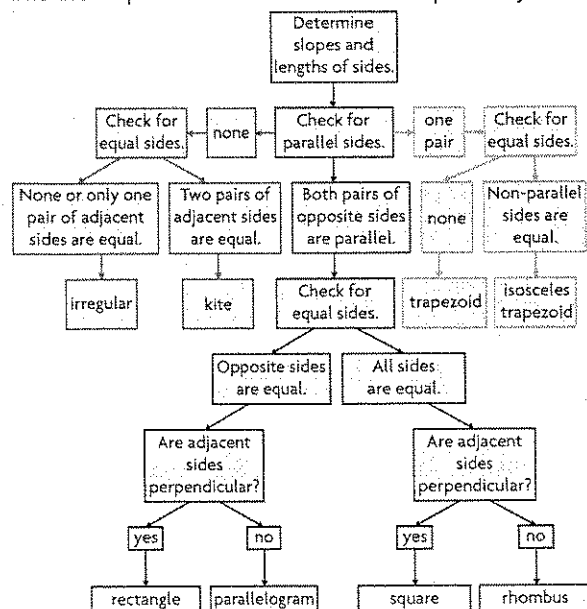
Because $PQRS$ is a rectangle, the change in coordinates from R to S must be the same as from Q to P if RS and QP are to be equal in length and parallel. The run from Q to P is $4 - 9 = -5$. The rise is $12 - 16 = -4$. Applying this change in coordinates to R , the coordinates of S are $(13 - 5, 8 - 4)$, or $(8, 4)$.

c) For example, by the symmetry of rectangles, predict that the diagonals are the same length. Determine the lengths of the diagonals, PR and QS using the distance formula.

$$\begin{aligned} PR &= \sqrt{(13-4)^2 + (8-12)^2} & QS &= \sqrt{(8-9)^2 + (4-16)^2} \\ &= \sqrt{81+16} & &= \sqrt{1+144} \\ &= \sqrt{97} & &= \sqrt{145} \\ &\approx 9.85 & &= \sqrt{145} \\ & & &\approx 12.04 \end{aligned}$$

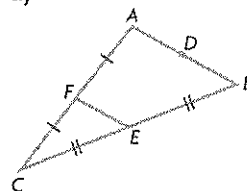
$PR = QS$. Therefore, the diagonals are equal in length.

19. Answers may vary, e.g., the slopes and side lengths allow you to classify the quadrilateral. These can be determined by substituting the coordinates of the vertices into the slope and distance formulas respectively.



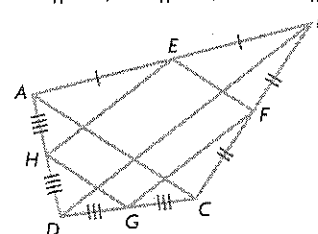
20. Answers may vary, e.g.,

a)



Since $\frac{AC}{FC} = \frac{BC}{EC}$, and $\angle ACB = \angle FCE$, then $\triangle ABC$ is similar to $\triangle FEC$.

Since F is a midpoint of AC and E is a midpoint of BC , the dimensions of $\triangle FEC$ are half those of $\triangle ABC$. Also, because of the similarity and lack of rotation, $FC \parallel AC$, $EC \parallel BC$, and $EF \parallel BA$.



b) $\triangle AHE$ is similar to $\triangle ADB$; therefore, $HE \parallel DB$. $\triangle CGF$ is similar to $\triangle CDB$. Therefore, $GF \parallel DB$. $\triangle DGH$ is similar to $\triangle DCA$. Therefore, $GH \parallel CA$. $\triangle BFE$ is similar to $\triangle BCA$. Therefore, $FE \parallel CA$. Since $HE \parallel DB \parallel GF$ and $GF \parallel CA \parallel FE$, then $EFGH$ is a parallelogram.

Lesson 2.5: Verifying Properties of Geometric Figures, page 109

1. Calculate the length of each line using the length formula.

$$\begin{aligned} d_{AC} &= \sqrt{(-6-1)^2 + (4-0)^2} & d_{BD} &= \sqrt{[-2-(-3)]^2 + [6-(-2)]^2} \\ &= \sqrt{49+16} & &= \sqrt{1+64} \\ &= \sqrt{65} & &= \sqrt{65} \\ &\approx 8.06 & &\approx 8.06 \end{aligned}$$

Therefore AC and BD are the same length.

2. To prove two lines are perpendicular, show that their slopes are negative reciprocals of each other.

$$\begin{aligned} m_{JL} &= \frac{6-2}{3-11} & m_{KM} &= \frac{8-(-4)}{9-3} \\ &= -\frac{4}{8} & &= \frac{12}{6} \\ &= -\frac{1}{2} & &= 2 \end{aligned}$$

The diagonals are perpendicular to each other.

3. Let S represent the midpoint of PR .

$$\begin{aligned} S &= \left(\frac{-2+5}{2}, \frac{1+2}{2} \right) \\ &= (1.5, 1.5) \end{aligned}$$

Since the median by definition passes through the midpoint of one side and the opposite vertex, then QS must bisect PR . To prove it is the perpendicular bisector, I must now show their slopes are negative reciprocals.

$$m_{QS} = \frac{5-1.5}{1-1.5} = -7$$

$$m_{PR} = \frac{1-2}{-2-5} = \frac{1}{7}$$

The median is also the right bisector of PR .

4. If the diagonals have the same midpoint, then they bisect each other.

$$M_{JL} = \left(\frac{10+(-12)}{2}, \frac{0+(-6)}{2} \right) = \left(\frac{-2}{2}, \frac{-6}{2} \right) = (-1, -3)$$

$$M_{KM} = \left(\frac{-8+6}{2}, \frac{6-12}{2} \right) = \left(\frac{-2}{2}, \frac{-6}{2} \right) = (-1, -3)$$

Therefore, $M_{JL} = M_{KM} = (-1, -3)$.

$$d_{AC} = \sqrt{(-6-8)^2 + [5-(-13)]^2}$$

$$= \sqrt{(-14)^2 + 18^2}$$

$$= \sqrt{196 + 324}$$

$$= \sqrt{520}$$

$$\approx 22.80$$

$$d_{BD} = \sqrt{[12-(-10)]^2 + [-1-(-7)]^2}$$

$$= \sqrt{22^2 + 6^2}$$

$$= \sqrt{484 + 36}$$

$$= \sqrt{520}$$

$$\approx 22.80$$

Since $AC = BD = 22.80$ units, then the diagonals are of equal length.

6. Conjecture: The diagonals of a rectangle $ABCD$ are equal in length. Using the Pythagorean theorem, $AC^2 = AB^2 + BC^2$ and $BD^2 = BC^2 + DC^2$; since $AB = DC$, then $AC^2 = BD^2$, so $AC = BD$. Therefore, the conjecture is true.

7. Conjecture: The diagonals of a kite $JKLM$ are perpendicular. Since $\triangle JKL$ and $\triangle JLM$ are isosceles, the median from K to JL is perpendicular bisector of JL and the median from M to JL is perpendicular bisector of JL . So both medians, and hence diagonal KM , lie along perpendicular bisector of JL . Therefore, the diagonals are perpendicular.

8. Begin by determining the slope of the median.

$$M_{EF} = \left(\frac{1+(-1)}{2}, \frac{8+(-2)}{2} \right) = \left(\frac{0}{2}, \frac{6}{2} \right) = (0, 3)$$

$$m_{\text{median}} = \frac{3-4}{0-(-5)} = -\frac{1}{5}$$

Both the altitude and the median pass through D . Now determine the slope of the altitude.

$$m_{EF} = \frac{8-(-2)}{1-(-1)} = \frac{10}{2} = 5$$

$$m_{\text{alt}} = -\frac{1}{5}$$

Since the altitude and the median go through the same point and have the same slope, they are the same line.

9. First determine all the midpoints.

$$M_{PQ} = \left(\frac{-2+0}{2}, \frac{-2+4}{2} \right) = (-1, 1) = A$$

$$M_{QR} = \left(\frac{0+6}{2}, \frac{4+3}{2} \right) = (3, 3.5) = B$$

$$M_{RS} = \left(\frac{6+8}{2}, \frac{3+(-1)}{2} \right) = (7, 1) = C$$

$$M_{SP} = \left(\frac{8+(-2)}{2}, \frac{-1+(-2)}{2} \right) = (3, -1.5) = D$$

Now calculate the lengths of each side.

$$d_{AB} = \sqrt{(-1-3)^2 + (1-3.5)^2} = \sqrt{(-4)^2 + (-2.5)^2} = \sqrt{16 + 6.25} = \sqrt{22.25} \approx 4.72$$

$$d_{BC} = \sqrt{(7-3)^2 + (1-3.5)^2} = \sqrt{4^2 + (-2.5)^2} = \sqrt{16 + 6.25} = \sqrt{22.25} \approx 4.72$$

$$d_{CD} = \sqrt{(7-3)^2 + [1-(-1.5)]^2} = \sqrt{4^2 + (2.5)^2} = \sqrt{16 + 6.25} = \sqrt{22.25} \approx 4.72$$

$$d_{DA} = \sqrt{(-1-3)^2 + [1-(-1.5)]^2} = \sqrt{(-4)^2 + (2.5)^2} = \sqrt{16 + 6.25} = \sqrt{22.25} \approx 4.72$$

Since all the sides are equal, then the joined midpoints form a rhombus.

10. First, determine all the midpoints.

$$M_{RS} = \left(\frac{-5+(-1)}{2}, \frac{2+3}{2} \right) = (-3, 2.5) = A$$

$$M_{ST} = \left(\frac{-1+(-2)}{2}, \frac{3+(-1)}{2} \right) = (-1.5, 1) = B$$

$$M_{TU} = \left(\frac{-2+(-6)}{2}, \frac{-1+(-2)}{2} \right) = (-4, -1.5) = C$$

$$M_{UR} = \left(\frac{-6+(-5)}{2}, \frac{-2+2}{2} \right) = (-5.5, 0) = D$$

Now check whether the diagonals are the same length.

$$\begin{aligned}d_{AC} &= \sqrt{[-3 - (-4)]^2 + [2.5 - (-1.5)]^2} \\&= \sqrt{1^2 + 4^2} \\&= \sqrt{1 + 16} \\&= \sqrt{17} \\&\approx 4.12\end{aligned}$$

$$\begin{aligned}d_{BD} &= \sqrt{[-1.5 - (-5.5)]^2 + (1 - 0)^2} \\&= \sqrt{4^2 + 1^2} \\&= \sqrt{16 + 1} \\&= \sqrt{17} \\&\approx 4.12\end{aligned}$$

Since the diagonals are the same length, then $ABCD$ is a rectangle.

11. To prove the diagonals bisect each other, show they share the same midpoint.

$$\begin{aligned}M_{RT} &= \left(\frac{-5 + (-2)}{2}, \frac{2 + (-1)}{2} \right) & M_{SU} &= \left(\frac{-1 + (-6)}{2}, \frac{3 + (-2)}{2} \right) \\&= (-3.5, 0.5) & &= (-3.5, 0.5)\end{aligned}$$

To prove they intersect perpendicularly, show that their slopes are negative reciprocals of each other.

$$\begin{aligned}m_{RT} &= \frac{2 - (-1)}{-5 - (-2)} & m_{SU} &= \frac{3 - (-2)}{-1 - (-6)} \\&= -1 & &= 1\end{aligned}$$

The diagonals of the rhombus bisect each other at a right angle.

12. First, determine all the midpoints.

$$\begin{aligned}M_{AB} &= \left(\frac{2 + (-10)}{2}, \frac{-12 + (-8)}{2} \right) \\&= (-4, -10) \\&= W\end{aligned}$$

$$\begin{aligned}M_{BC} &= \left(\frac{-10 + (-6)}{2}, \frac{-8 + 4}{2} \right) \\&= (-8, -2) \\&= X\end{aligned}$$

$$\begin{aligned}M_{CD} &= \left(\frac{-6 + 6}{2}, \frac{4 + 0}{2} \right) & M_{DA} &= \left(\frac{6 + 2}{2}, \frac{0 + (-12)}{2} \right) \\&= (2, 0) & &= (4, -6) \\&= Y & &= Z\end{aligned}$$

Now examine the length of the diagonals to see if the shape is a rectangle.

$$\begin{aligned}d_{WY} &= \sqrt{(-4 - 0)^2 + (-10 - 2)^2} \\&= \sqrt{(-4)^2 + (-12)^2} \\&= \sqrt{16 + 144} \\&= \sqrt{160} \\&\approx 12.65\end{aligned}$$

$$\begin{aligned}d_{XZ} &= \sqrt{(-8 - 4)^2 + [-2 - (-6)]^2} \\&= \sqrt{(-12)^2 + 4^2} \\&= \sqrt{144 + 16} \\&= \sqrt{160} \\&\approx 12.65\end{aligned}$$

Rectangles have two pairs of identical sides. If these pairs are also equal, then the rectangle is also a square.

$$\begin{aligned}d_{WX} &= \sqrt{[-4 - (-8)]^2 + [-10 - (-2)]^2} \\&= \sqrt{4^2 + (-8)^2} \\&= \sqrt{16 + 64} \\&= \sqrt{80} \\&\approx 8.94\end{aligned}$$

$$\begin{aligned}d_{YZ} &= \sqrt{(-8 - 0)^2 + (-2 - 2)^2} \\&= \sqrt{(-8)^2 + (-4)^2} \\&= \sqrt{64 + 16} \\&= \sqrt{80} \\&\approx 8.94\end{aligned}$$

The shape formed by the midpoints of the square is another square.

13. a) Show the points lie on the circle by substituting in the values and solving.

$$\begin{aligned}x^2 + y^2 &= (-4)^2 + (3)^2 \\x^2 + y^2 &= 9 + 16 \\x^2 + y^2 &= 25 \\x^2 + y^2 &= 3^2 + (-4)^2 \\x^2 + y^2 &= 9 + 16 \\x^2 + y^2 &= 25\end{aligned}$$

b) The centre of the circle is the origin. Begin by determining the equation of the perpendicular bisector and then determine if the origin is a point on the line.

$$\begin{aligned}M_{AB} &= \left(\frac{-4 + 3}{2}, \frac{3 + (-4)}{2} \right) & m_{AB} &= \frac{3 - (-4)}{-4 - 3} \\&= (-0.5, -0.5) & &= -1 \\& & m_{\text{bisector}} &= 1\end{aligned}$$

Equation of the perpendicular bisector:

$$\begin{aligned}y &= mx + b \\(-0.5) &= (-0.5)(1) + b \\b &= 0 \\y &= x\end{aligned}$$

When $x = 0$ then $y = (0) = 0$, so centre $(0, 0)$ of circle lies on the perpendicular bisector.

14. a) Determine the midpoints of BC and AD and then calculate the slopes of this line, AB , and DC .

$$\begin{aligned}M_{BC} &= \left(\frac{-2 + (-4)}{2}, \frac{1 + (-2)}{2} \right) & M_{AD} &= \left(\frac{1 + 2}{2}, \frac{2 + 0}{2} \right) \\&= (-3, -0.5) & &= (1.5, 1) \\m_{M_{BC}M_{AD}} &= \frac{-0.5 - 1}{-3 - 1.5} & m_{AB} &= \frac{2 - 1}{1 - (-2)} & m_{DC} &= \frac{0 + 2}{2 - (-4)} \\&= \frac{1}{3} & &= \frac{1}{3} & &= \frac{1}{3}\end{aligned}$$

Since all three lines have the same slope, they must be parallel to each other.

b) Determine the lengths of all the required sides.

$$\begin{aligned}d_{\text{segment}} &= \sqrt{(-3-1.5)^2 + (-0.5-1)^2} \\&= \sqrt{20.25 + 2.25} \\&= \sqrt{22.50} \\&\approx 4.74\end{aligned}$$

$$\begin{aligned}d_{BC} &= \sqrt{[1-(-2)]^2 + (2-1)^2} & d_{AD} &= \sqrt{[2-(-4)]^2 + [0-(-2)]^2} \\&= \sqrt{9+1} & &= \sqrt{36+4} \\&= \sqrt{10} & &= \sqrt{40}\end{aligned}$$

Now check that the length of the segment is half the sum of the parallel sides.

$$\text{sum} = \frac{BC + AD}{2}$$

$$\text{sum} = \frac{\sqrt{10} + \sqrt{40}}{2}$$

$$\text{sum} \approx 4.74$$

$$\text{sum} = \text{Segment}$$

15. Answers may vary, e.g., I recognize that this triangle has a vertical edge along $y = 0$. Since that edge goes from -2 to 5 , then the base of the rectangle will be 7 units. If BC is the base then A must be the vertex which the height crosses through. The height goes from 0 to 4 , so the height is 4 units.

$$\begin{aligned}\text{Area of } \triangle ABC &= \frac{1}{2}(7)(4) \\&= 14\end{aligned}$$

Now determine the vertices of the smaller triangle.

$$\begin{aligned}M_{AB} &= \left(\frac{3+(-2)}{2}, \frac{4+0}{2} \right) & M_{BC} &= \left(\frac{-2+5}{2}, \frac{0+0}{2} \right) \\&= (0.5, 2) & &= (1.5, 0)\end{aligned}$$

$$\begin{aligned}M_{CA} &= \left(\frac{5+3}{2}, \frac{0+4}{2} \right) \\&= (4, 2)\end{aligned}$$

Using the same logic as above, the base is 3.5 units and the height is 2 units.

$$\begin{aligned}\text{Area of } \triangle M_{AB}M_{BC}M_{AC} &= \frac{1}{2}(3.5)(2) \\&= 3.5 \\&= \frac{1}{4}(14)\end{aligned}$$

The area of the smaller triangle is a quarter of the area of the larger triangle.

16. Answers may vary, e.g.,

Determine the midpoint of the hypotenuse using the midpoint formula.

Determine the distance from the midpoint to each vertex using the distance formula.

Compare the distances.

17. Answers may vary, e.g., let vertices of square be $A(0, 0)$, $B(2a, 0)$, $C(2a, 2a)$, $D(0, 2a)$. Calculate the midpoint of the sides and the diagonals.

$$\begin{aligned}M_{AB} &= \left(\frac{0+2a}{2}, \frac{0+0}{2} \right) & M_{BC} &= \left(\frac{2a+2a}{2}, \frac{0+2a}{2} \right) \\&= (a, 0) & &= (2a, a) \\&= W & &= X\end{aligned}$$

$$\begin{aligned}M_{CD} &= \left(\frac{2a+0}{2}, \frac{2a+2a}{2} \right) & M_{DA} &= \left(\frac{0+0}{2}, \frac{2a+0}{2} \right) \\&= (a, 2a) & &= (0, a) \\&= Y & &= Z\end{aligned}$$

$$\begin{aligned}M_{WY} &= \left(\frac{a+a}{2}, \frac{2a+0}{2} \right) & M_{XZ} &= \left(\frac{2a+0}{2}, \frac{a+a}{2} \right) \\&= (a, a) & &= (a, a)\end{aligned}$$

$$\begin{aligned}M_{AC} &= \left(\frac{0+2a}{2}, \frac{0+2a}{2} \right) & M_{BD} &= \left(\frac{0+2a}{2}, \frac{2a+0}{2} \right) \\&= (a, a) & &= (a, a)\end{aligned}$$

The midpoints of WY , XZ , AC , and BD are all (a, a) .

Lesson 2.6: Exploring Properties of Geometric Figures, page 113

1. Determine the point where the right bisectors meet since it is equal distance from all the vertices.

The right bisector of AB :

$$\begin{aligned}M_{AB} &= \left(\frac{5+(-2)}{2}, \frac{1+0}{2} \right) & m_{AB} &= \frac{1-0}{5-(-2)} \\&= \left(\frac{3}{2}, \frac{1}{2} \right) & &= \frac{1}{7}\end{aligned}$$

$$m_b = -7$$

The equation is

$$y = -7x + b$$

$$\frac{1}{2} = -7\left(\frac{3}{2}\right) + b$$

$$\frac{22}{2} = b$$

$$b = 11$$

$$y = -7x + 11$$

The right bisector of BC :

$$M_{BC} = \left(\frac{-2+4}{2}, \frac{0+8}{2} \right) = (1, 4)$$

$$m_{BC} = \frac{0-8}{-2-4} = \frac{8}{6} = \frac{4}{3}$$

The equation is

$$y = -\frac{3}{4}x + b$$

$$4 = -\frac{3}{4}(1) + b$$

$$b = \frac{19}{4}$$

$$y = -\frac{3}{4}x + \frac{19}{4}$$

Solving the system of equations:

$$-7x + 11 = -\frac{3}{4}x + \frac{19}{4}$$

$$-28x + 44 = -3x + 19$$

$$-25x = -25$$

$$x = 1$$

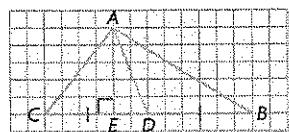
$$y = -7x + 11$$

$$y = -7(1) + 11$$

$$y = 4$$

Point $(1, 4)$ is the same distance from each vertex.

2. Answers may vary, e.g.,



Let AD represent the median and let AE represent the altitude. Since both $\triangle ACD$ and $\triangle ABD$ have the same base, $CD = DB$, and the same height, AE then they have the same area.

3. a) Calculate the lengths of all the line segments using the distance formula.

$$d_{AE} = \sqrt{(2-5)^2 + (9-0)^2} = \sqrt{9+81} = \sqrt{90}$$

$$d_{EB} = \sqrt{(7-5)^2 + (-6-0)^2} = \sqrt{4+36} = \sqrt{40}$$

$$d_{CE} = \sqrt{(7-5)^2 + (6-0)^2} = \sqrt{4+36} = \sqrt{40}$$

$$d_{ED} = \sqrt{(2-5)^2 + (-9-0)^2} = \sqrt{9+81} = \sqrt{90}$$

Using the products of the lengths:

$$AE \times EB = CE \times ED$$

$$= \sqrt{40} \times \sqrt{90}$$

$$= \sqrt{3600}$$

$$= 60$$

b) $AM \times MB = HM \times MX$

$$1 \times 1 = 0.5 \times MX$$

$$MX = 2$$

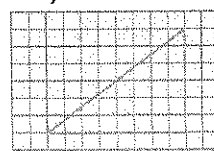
The diameter is

$$HM + MX = 0.5 + 2$$

$$= 2.5$$

The radius is equal to half the diameter, therefore the radius is 1.25 m.

4. a)



b) Answers may vary, e.g., this triangle has side lengths 6 cm, 8 cm, and 10 cm.

Similar Figures	Diagram	$A_1 = \text{Area on 6 cm side (cm}^2\text{)}$	$A_2 = \text{Area on 8 cm side (cm}^2\text{)}$	$A_1 + A_2 \text{ (cm}^2\text{)}$	Area on hypotenuse (cm ²)
square		$6^2 = 36$	$8^2 = 64$	$36 + 64 = 100$	$10^2 = 100$
semicircle		$0.5\pi(3)^2 = 14.14$	$0.5\pi(4)^2 = 25.13$	$14.14 + 25.13 = 39.27$	$0.5\pi(5)^2 = 39.27$
rectangle		$6 \times 3 = 18$	$8 \times 4 = 32$	$18 + 32 = 50$	$10 \times 5 = 50$
equilateral triangle		$0.5(6)(5.196) = 15.59$	$0.5(8)(6.928) = 27.71$	$15.59 + 27.71 = 43.30$	$0.5(10)(8.660) = 43.30$
right triangle		$0.5(6)(3) = 9$	$0.5(8)(4) = 16$	$9 + 16 = 25$	$0.5(10)(5) = 25$
parallelogram		$0.5(6+6)3 = 18$	$0.5(8+8)4 = 32$	$18 + 32 = 50$	$0.5(10+10)5 = 50$

- c) The dimensions of the right angle triangle does not have an effect.
d) The sum of the two smaller areas always equals the larger area.

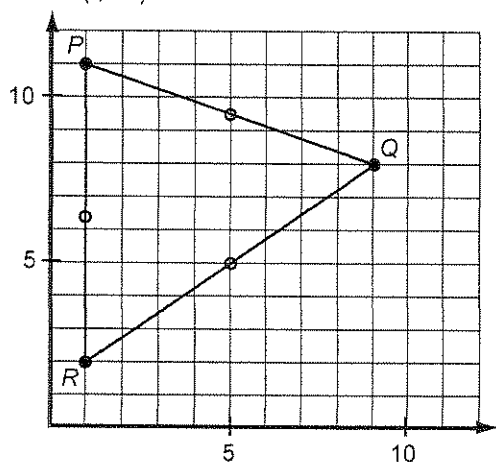
Curious Math, page 114

1.

$$M_{PQ} = \left(\frac{1+9}{2}, \frac{11+8}{2} \right) = (5, 9.5)$$

$$M_{QR} = \left(\frac{1+9}{2}, \frac{2+8}{2} \right) = (5, 5)$$

$$M_{RP} = \left(\frac{1+1}{2}, \frac{11+2}{2} \right) = (1, 6.5)$$



blue

2. The altitude of PQ:

$$m_{PQ} = \frac{11-8}{1-9} = -\frac{3}{8}$$

$$m_{alt} = \frac{8}{3}$$

Using the negative reciprocal slope and point R, I can determine the equation of the altitude.

$$y = \frac{8}{3}x + b$$

$$2 = \frac{8}{3}(1) + b$$

$$b = -\frac{2}{3}$$

$$y = \frac{8}{3}x - \frac{2}{3}$$

Now determine the point at which it crosses PQ. To do so, I will need the equation of PQ; then I will determine their intersection.

$$y = -\frac{3}{8}x + b$$

$$11 = -\frac{3}{8}(1) + b$$

$$b = \frac{91}{8}$$

$$y = -\frac{3}{8}x + \frac{91}{8}$$

Solve:

$$\begin{aligned} -\frac{3}{8}x + \frac{91}{8} &= \frac{8}{3}x - \frac{2}{3} \\ -\frac{9}{24}x + \frac{273}{24} &= \frac{64}{24}x - \frac{16}{24} \\ \frac{289}{24} &= \frac{73}{24}x \\ 289 &= 73x \\ \frac{289}{73} &= x \end{aligned}$$

$$\begin{aligned} y &= \frac{8}{3}x - \frac{2}{3} \\ y &= \frac{8}{3}\left(\frac{289}{73}\right) - \frac{2}{3} \\ y &= \frac{2312}{219} - \frac{2}{3} \\ y &= \frac{6936}{657} - \frac{438}{657} \\ y &= \frac{6498}{657} \end{aligned}$$

The point on the circle is (3.96, 9.89).

The altitude of QR:

$$m_{QR} = \frac{8-2}{9-1}$$

$$= \frac{6}{8}$$

$$= \frac{3}{4}$$

$$m_{alt} = -\frac{4}{3}$$

Using the negative reciprocal slope and point P, I can determine the equation of the altitude.

$$y = -\frac{4}{3}x + b$$

$$11 = -\frac{4}{3}(1) + b$$

$$b = \frac{37}{3}$$

$$y = -\frac{4}{3}x + \frac{37}{3}$$

Now to determine at which point the altitude crosses QR. To do so, I will need the equation of QR, and I will determine their intersection.

$$y = \frac{3}{4}x + b$$

$$2 = \frac{3}{4}(1) + b$$

$$b = \frac{5}{4}$$

$$y = \frac{3}{4}x + \frac{5}{4}$$

Solve:

$$\frac{3}{4}x + \frac{5}{4} = -\frac{4}{3}x + \frac{37}{3}$$

$$\frac{25}{12}x = \frac{133}{12}$$

$$x = \frac{133}{25}$$

$$y = -\frac{4}{3}x + \frac{37}{3}$$

$$y = -\frac{4}{3}\left(\frac{133}{25}\right) + \frac{37}{3}$$

$$y = \frac{131}{25}$$

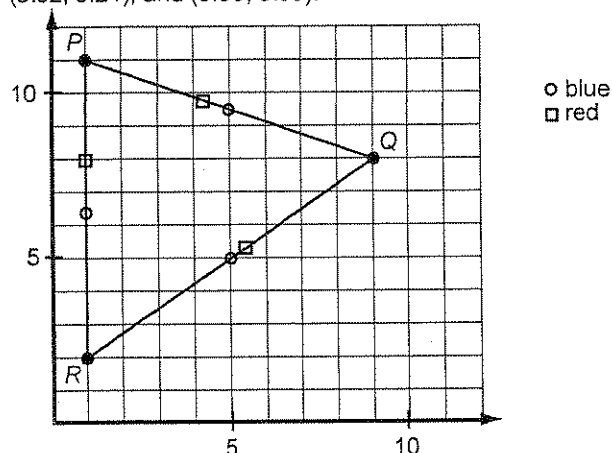
The point on the circle is (5.32, 5.24). The altitude of PR:

$$m_{PR} = \frac{11-2}{1-1}$$

$$= \frac{9}{0}$$

$$m_{alt} = \frac{0}{9}$$

PR is a vertical line and its median is a horizontal line. P and R have x -coordinates of 1, therefore the equation of the vertical line is $x = 1$. The horizontal line goes through Q , therefore the equation of that line is $y = 8$. The point of intersection must be $(1, 8)$. The three red points are $(1, 8)$, $(5.32, 5.24)$, and $(3.96, 9.89)$.



3. The equations of the altitudes are:

$$1) y = \frac{8}{3}x - \frac{2}{3} \quad 2) y = -\frac{4}{3}x + \frac{37}{3} \quad 3) y = 8$$

Determine the orthocentre by substituting the value of y from equation 3 into equation 1.

$$8 = \frac{8}{3}x - \frac{2}{3}$$

$$\frac{24}{3} = \frac{8}{3}x - \frac{2}{3}$$

$$\frac{26}{3} = \frac{8}{3}x$$

$$x = \frac{26}{8}$$

$$x = \frac{13}{4}$$

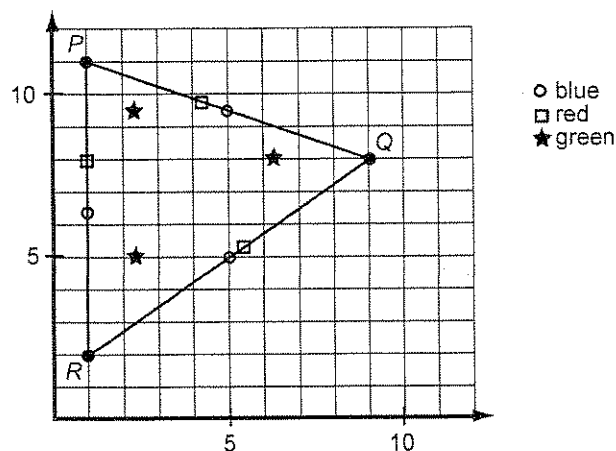
The orthocentre is $(3.25, 8)$. Now calculate the midpoints between O and the vertices.

$$M_{OP} = \left(\frac{3.25+1}{2}, \frac{8+11}{2} \right) = (2.125, 9.5)$$

$$M_{OQ} = \left(\frac{3.25+9}{2}, \frac{8+8}{2} \right) = (6.125, 8)$$

$$M_{OR} = \left(\frac{3.25+1}{2}, \frac{8+2}{2} \right) = (2.125, 5)$$

The green points are $(2.125, 9.5)$, $(6.125, 8)$, and $(2.125, 5)$.



4. The midpoints and the slopes of all the lines were calculated in the previous questions.

Perpendicular bisector of QR : RP :

$$\begin{aligned} \text{slope} &= -\frac{4}{3} & \text{slope} &= 0 \\ \text{midpoint} &= (5, 5) & \text{midpoint} &= (1, 6.5) \\ & & y &= (0)x + b \\ & & 6.5 &= b \\ & & \text{Equation: } &y = 6.5 \end{aligned}$$

$$y = -\frac{4}{3}x + b$$

$$5 = -\frac{4}{3}(5) + b$$

$$b = \frac{35}{3}$$

$$y = -\frac{4}{3}x + \frac{35}{3}$$

Solve the system of equations:

$$-\frac{4}{3}x + \frac{35}{3} = 6.5$$

$$-\frac{4}{3}x = \frac{19.5}{3} - \frac{35}{3}$$

$$-4x = 19.5 - 35$$

$$-4x = -15.5$$

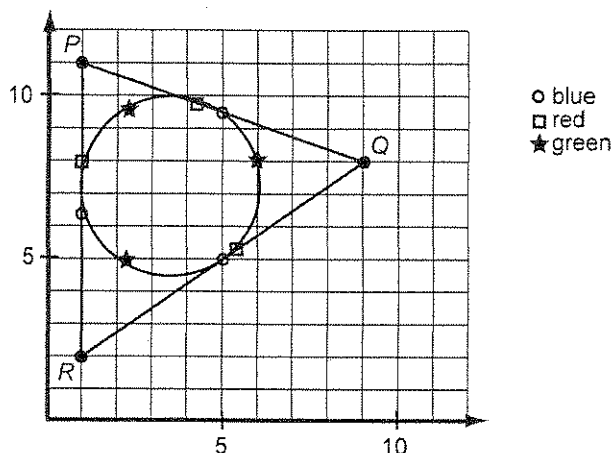
$$x = 3.875$$

$$y = 6.5$$

The circumcentre is $(3.875, 6.5)$.

Now determine the midpoint of the line joining the orthocentre and the circumcentre.

$$\begin{aligned} \text{mid}_{co} &= \left(\frac{3.875+3.25}{2}, \frac{6.5+8}{2} \right) \\ &= (3.5625, 7.25) \end{aligned}$$



5. As shown in the steps above, I can determine the points for any triangle. Three of the points will be the midpoints of each side, the next three points will be where the altitudes meet the side of the triangle, and the final three points are the midpoints between the points of the triangle and the orthocentre.

Lesson 2.7: Using Coordinates to Solve Problems, page 120

$$1. a) m_{BC} = \frac{-2-1}{-1-5} = \frac{-3}{-6} = \frac{1}{2}$$

$$b) m_{BC} = \frac{1}{2} \\ m_{AD} = -\frac{1}{\left(\frac{1}{2}\right)} = -2$$

c) Use the slope of -2 and point A to determine the equation of the line containing AD .

$$y = mx + b \\ 4 = -2(-1) + b \\ b = 2$$

The equation is $y = -2x + 2$.

2. Use the slope of $\frac{1}{2}$ and point C to determine the equation of the line containing BC .

$$y = mx + b$$

$$1 = \frac{1}{2}(5) + b$$

$$b = -\frac{3}{2}$$

The equation is $y = \frac{1}{2}x - \frac{3}{2}$.

3. To determine the coordinates of D , solve the system formed by the equations from parts a) and b).

$$y = \frac{1}{2}x - \frac{3}{2}$$

$$y = -2x + 2$$

Substitute and solve:

$$-2x + 2 = \frac{1}{2}x - \frac{3}{2}$$

$$\frac{5}{2}x = \frac{7}{2}$$

$$x = \frac{7}{5}$$

Solve for the y -coordinate:

$$y = -2x + 2$$

$$y = -2\left(\frac{7}{5}\right) + 2$$

$$y = -\frac{14}{5} + 2$$

$$y = -\frac{4}{5}$$

The coordinates of D are $\left(\frac{7}{5}, -\frac{4}{5}\right)$.

4. Length of BC :

$$d_{BC} = \sqrt{[5 - (-1)]^2 + [1 - (-2)]^2} \\ = \sqrt{36 + 9} \\ = \sqrt{45} \\ \approx 6.71$$

Length of AD :

$$d_{AD} = \sqrt{\left(-1 - \frac{7}{5}\right)^2 + \left[4 - \left(-\frac{4}{5}\right)\right]^2} \\ = \sqrt{\left(-\frac{12}{5}\right)^2 + \left(\frac{24}{5}\right)^2} \\ = \sqrt{\frac{144}{25} + \frac{576}{25}} \\ = \sqrt{\frac{720}{25}} \\ \approx 5.37$$

5.

$$A = \frac{1}{2}bh \\ = \frac{1}{2}\sqrt{45}\sqrt{\frac{720}{25}} \\ = \frac{1}{2}\sqrt{1296} \\ = 18$$

The triangle has an area of 18 square units.

6. a) Begin by calculating the midpoint of BC .

$$M_{BC} = \left(\frac{-5+5}{2}, \frac{-6+0}{2}\right) \\ = (0, -3)$$

Now, using $(0, -3)$ and point A , I can determine the equation of the median.

Calculate the slope:

$$m_{\text{median}} = \frac{-3-2}{0-(-3)} \\ = -\frac{5}{3}$$

Calculate the b -value:

$$-3 = -\frac{5}{3}(0) + b \\ b = -3$$

The equation is $y = -\frac{5}{3}x - 3$.

b) Determine the slope of BC .

$$m_{BC} = \frac{0 - (-6)}{5 - (-5)} \\ = \frac{3}{5}$$

Since the slope of BC is the negative reciprocal of the slope of the median, then the median and altitude must have the same equation, $y = -\frac{5}{3}x - 3$.

c) The line represented by $y = -\frac{5}{3}x - 3$ is also the right bisector since it crosses BC at the midpoint, while forming a right angle.

d) The triangle is isosceles since the median and altitude are the same.

7. a) Since these points are the end points of the diameter, their midpoint will be the centre of the circle.

$$M = \left(\frac{-9 + 9}{2}, \frac{2 + (-2)}{2} \right) \\ = (0, 0)$$

The equation is of the form $x^2 + y^2 = r^2$. Now I will solve for r^2 .

$$r^2 = x^2 + y^2 \\ r^2 = (-9)^2 + (2)^2 \\ r^2 = 81 + 4 \\ r^2 = 85$$

The equation is $x^2 + y^2 = 85$.

b) Check by substituting in the point.

$$x^2 + y^2 = 7^2 + 6^2 \\ x^2 + y^2 = 49 + 36 \\ x^2 + y^2 = 85$$

Point $(7, 6)$ is on the circle.

c)

$$m_{PR} = \frac{2 - 6}{-9 - 7} \quad m_{RQ} = \frac{6 - (-2)}{7 - 9} \quad m_{PR} = -\frac{1}{m_{RQ}} \\ = \frac{4}{16} \quad = -\frac{8}{2} \\ = \frac{1}{4} \quad = -4$$

8. Determine the length of the base of the triangle (LM).

$$d_{LM} = \sqrt{(3 - 4)^2 + [4 - (-3)]^2} \\ = \sqrt{(-1)^2 + 7^2} \\ = \sqrt{1 + 49} \\ = \sqrt{50}$$

Now determine the equations for the base and altitude.

Base:

$$m_{LM} = \frac{4 - (-3)}{3 - 4} \\ = -7 \\ y = mx + b \\ 4 = -7(3) + b \\ b = 25 \\ y = -7x + 25$$

Altitude: The slope is the negative reciprocal of the

$$\text{base, } m_A = \frac{1}{7}.$$

$$y = mx + b \\ -1 = \frac{1}{7}(-4) + b \\ b = -\frac{3}{7} \\ y = \frac{1}{7}x - \frac{3}{7}$$

Solve the system of equations to calculate the intersection.

$$\frac{1}{7}x - \frac{3}{7} = -7x + 25 \quad y = \frac{1}{7}x - \frac{3}{7} \\ \frac{50}{7}x = \frac{178}{7} \quad y = \frac{1}{7}\left(\frac{89}{25}\right) - \frac{3}{7} \\ x = \frac{89}{25} \quad y = \frac{2}{25}$$

Now calculate the height using $\left(\frac{89}{25}, \frac{2}{25}\right)$ and $(-4, -1)$.

$$d_h = \sqrt{\left[\frac{89}{25} - (-4)\right]^2 + \left[\frac{2}{25} - (-1)\right]^2} \\ = \sqrt{\frac{35721}{625} + \frac{729}{625}} \\ = \sqrt{\frac{36450}{625}} \\ = \sqrt{58.32}$$

To calculate the area of a triangle, multiply half the base by the height.

$$A = \frac{1}{2}bh \\ A = \frac{1}{2}\sqrt{50}\sqrt{58.32} \\ A = 27$$

The area is 27 square units.

9. To determine the equation of an altitude, I need the slope perpendicular to the base and the opposite point.

Altitude passing through D : Altitude passing through F :

$$m_{EF} = \frac{2 - 2}{6 - (-3)} \quad m_{DE} = \frac{8 - 2}{2 - 6} \\ = 0 \quad = -\frac{6}{4} \\ \text{The perpendicular slope is} \\ \text{undefined; this means that} \\ \text{the line is vertical. The line} \\ \text{must pass through } D(2, 8), \\ \text{therefore the equation is} \\ x = 2. \quad m_F = \frac{2}{3}$$

Now determine the equation with the F slope and point F .

$$y = mx + b$$

$$2 = \frac{2}{3}(-3) + b$$

$$2 = -2 + b$$

$$b = 4$$

$$y = \frac{2}{3}x + 4$$

The intersection must occur when $x = 2$.

$$y = \frac{2}{3}x + 4$$

$$y = \frac{2}{3}(2) + 4$$

$$y = \frac{4}{3} + 4$$

$$y = \frac{16}{3}$$

The point is $\left(2, \frac{16}{3}\right)$.

10. Calculate the midpoints of PQ and QR .

$$M_{PQ} = \left(\frac{-12+4}{2}, \frac{6+0}{2}\right) \quad M_{QR} = \left(\frac{4+(-8)}{2}, \frac{0+(-6)}{2}\right)$$

$$= (-4, 3)$$

$$= (-2, -3)$$

The slope of the median passing through R is

$$m = \frac{-6-3}{-8-(-4)}$$

$$= \frac{9}{4}$$

The equation is

$$y = \frac{9}{4}x + b$$

$$-6 = \frac{9}{4}(-8) + b$$

$$-6 = -18 + b$$

$$b = 12$$

$$y = \frac{9}{4}x + 12$$

The equation is

$$y = -\frac{9}{10}x + b$$

$$6 = -\frac{9}{10}(-12) + b$$

$$b = -\frac{24}{5}$$

$$y = -\frac{9}{10}x - \frac{24}{5}$$

Solve the system:

$$\frac{9}{4}x + 12 = -\frac{9}{10}x - \frac{24}{5}$$

$$\frac{63}{20}x = -\frac{84}{5}$$

$$x = -\frac{16}{3}$$

The slope of the median passing through P is

$$m_p = \frac{6-(-3)}{-12-(-2)}$$

$$= -\frac{9}{10}$$

Solve for the y -value.

$$y = \frac{9}{4}x + 12$$

$$y = \frac{9}{4}\left(-\frac{16}{3}\right) + 12$$

$$y = -12 + 12$$

$$y = 0$$

The coordinates of the centroid are $\left(-\frac{16}{3}, 0\right)$.

11. Determine the equation of two right bisectors of the triangle and then calculate their point of intersection. To determine the equation, calculate the midpoint of a side and its slope, then use the negative reciprocal slope and this point to determine the equation.

Perpendicular bisector of JK :

$$m_{JK} = \frac{0-8}{-2-2} \quad M_{JK} = \left(\frac{-2+2}{2}, \frac{0+8}{2}\right)$$

$$= \frac{8}{-4} \quad = (0, 4)$$

$$= -2$$

$$m_B = -\frac{1}{2}$$

The equation:

$$y = -\frac{1}{2}x + b$$

$$4 = -\frac{1}{2}(0) + b$$

$$b = 4$$

$$y = -\frac{1}{2}x + 4$$

Perpendicular bisector of JL :

$$m_{JL} = \frac{3-0}{7-(-2)}$$

$$= \frac{3}{9}$$

$$= \frac{1}{3}$$

$$m_B = -3$$

The equation:

$$y = -3x + b$$

$$\frac{3}{2} = -3\left(\frac{5}{2}\right) + b$$

$$\frac{3}{2} = -\frac{15}{2} + b$$

$$b = 9$$

$$y = -3x + 9$$

Solve the system to determine the circumcentre:

$$-3x + 9 = -\frac{1}{2}x + 4$$

$$-\frac{5}{2}x = -5$$

$$x = 2$$

$$y = -\frac{1}{2}(2) + 4$$

$$y = -1 + 4$$

$$y = 3$$

The circumcentre is $(2, 3)$.

12. The right bisectors' intersection is an equal distance from each vertex. I will determine the equations of AB 's and AC 's right bisectors.

AB:

$$m_{AB} = \frac{2-6}{2-10}$$

$$= \frac{-4}{-8}$$

$$= \frac{1}{2}$$

$$m_b = -2$$

$$M_{AB} = \left(\frac{2+10}{2}, \frac{2+6}{2} \right)$$

$$= (6, 4)$$

AC:

$$m_{AC} = \frac{2-8}{2-4}$$

$$= \frac{-6}{-2}$$

$$= 3$$

$$m_b = -\frac{1}{3}$$

$$M_{AC} = \left(\frac{2+4}{2}, \frac{2+8}{2} \right)$$

$$= (3, 5)$$

Solve the system of equations:

$$-\frac{1}{3}x + 6 = -2x + 16$$

$$-x + 18 = -6x + 48$$

$$5x = 30$$

$$x = 6$$

Solve for the y-value.

$$y = -2x + 16$$

$$y = -2(6) + 16$$

$$y = -12 + 16$$

$$y = 4$$

The tennis court should be at (6, 4).

13. e.g., i) I could calculate the length of DC, EC and FC then check they are equal.

ii) I could construct the perpendicular bisectors for $\triangle DEF$ and check that their intersection is at C.

14. The centre of mass is where all the medians intersect. For simplicity, I will label the points A(8, 12), B(12, 4) and C(2, 8). I will begin by determining the equation of the median passing through A. I need to know the midpoint of BC.

$$M_{BC} = \left(\frac{12+2}{2}, \frac{4+8}{2} \right)$$

$$= (7, 6)$$

The slope is:

$$m = \frac{6-12}{7-8}$$

$$= \frac{-6}{-1}$$

$$= 6$$

Solve for the b-value:

$$y = -2x + b$$

$$4 = -2(6) + b$$

$$4 = -12 + b$$

$$b = 16$$

The equation is
 $y = -2x + 16$.

Solve for the b-value:

$$y = -\frac{1}{3}x + b$$

$$5 = -\frac{1}{3}(3) + b$$

$$5 = -1 + b$$

$$b = 6$$

The equation is
 $y = -\frac{1}{3}x + 6$.

The equation is:

$$y = mx + b$$

$$6 = 6(7) + b$$

$$b = -36$$

$$y = 6x - 36$$

First determine the equation of the median passing through B. I need to know the midpoint of CA.

$$M_{CA} = \left(\frac{2+8}{2}, \frac{8+12}{2} \right)$$

$$= (5, 10)$$

The equation is

$$m = \frac{10-4}{5-12}$$

$$= -\frac{6}{7}$$

$$y = -\frac{6}{7}x + b$$

$$10 = -\frac{6}{7}(5) + b$$

$$b = \frac{100}{7}$$

$$y = -\frac{6}{7}x + \frac{100}{7}$$

Solve the system of equations.

$$6x - 36 = -\frac{6}{7}x + \frac{100}{7}$$

$$42x - 252 = -6x + 100$$

$$48x = 352$$

$$x = \frac{22}{3}$$

Solve for the y-value.

$$y = 6x - 36$$

$$y = 6\left(\frac{22}{3}\right) - 36$$

$$y = 44 - 36$$

$$y = 8$$

The centre of mass is at $\left(\frac{22}{3}, 8\right)$.

15. Calculate the slope of each side and of the line joining each midpoint. Then prove each side is parallel to the line formed by the midpoints of the other two sides.

$$m_{AB} = \frac{-2-1}{-1-(-2)} \quad m_{BC} = \frac{1-0}{-2-5} \quad m_{CA} = \frac{0-(-2)}{5-(-1)}$$

$$= -3 \quad = -\frac{1}{7} \quad = \frac{2}{6}$$

$$= \frac{1}{3}$$

The midpoints of the triangle are

Point D Point E

$$M_{AB} = \left(\frac{-1+(-2)}{2}, \frac{-2+1}{2} \right) \quad M_{BC} = \left(\frac{-2+5}{2}, \frac{1+0}{2} \right)$$

$$= \left(-\frac{3}{2}, -\frac{1}{2} \right) \quad = \left(\frac{3}{2}, \frac{1}{2} \right)$$

Point F

$$M_{CA} = \left(\frac{5+(-1)}{2}, \frac{0+(-2)}{2} \right)$$

$$= (2, -1)$$

Now calculate the slopes of the small triangle's sides.

$$\begin{aligned}
 m_{DE} &= \frac{-\frac{1}{2} - \frac{1}{2}}{\frac{3}{2} - \frac{3}{2}} = \frac{-1}{0} = \text{undefined} \\
 m_{EF} &= \frac{\frac{1}{2} - (-1)}{\frac{3}{2} - 2} = \frac{\frac{3}{2}}{-\frac{1}{2}} = -3 \\
 m_{FD} &= \frac{-1 - \left(-\frac{1}{2}\right)}{2 - \left(-\frac{3}{2}\right)} = \frac{-\frac{1}{2}}{\frac{7}{2}} = -\frac{1}{7}
 \end{aligned}$$

Since the base is parallel to the joined midpoints, I can use similar triangle properties. The smaller triangle is in a 1:2 ratio with the bigger triangle. From here, I can deduce that the four small triangles are congruent. Three of the triangles are blue and one is green; this gives a 3:1 ratio.

16. a) To determine the shortest distance from each house to the pipeline, use a line perpendicular to the pipeline that passes through each house.

First, I need the equation of the pipeline. Using the slope and point G the equation is:

$$\begin{aligned}
 m_{GH} &= \frac{42 - (-30)}{38 - (-16)} = \frac{72}{54} = \frac{4}{3} \\
 m_p &= -\frac{3}{4} \\
 y &= mx + b \\
 42 &= \frac{4}{3}(38) + b \\
 b &= -\frac{26}{3} \\
 y &= \frac{4}{3}x - \frac{26}{3}
 \end{aligned}$$

To calculate the length of pipe required to join house A, I need to write an equation for the pipe.

$$\begin{aligned}
 y &= -\frac{3}{4}x + b \\
 32 &= -\frac{3}{4}(-16) + b \\
 b &= 20 \\
 y &= -\frac{3}{4}x + 20
 \end{aligned}$$

To calculate the point of intersection, solve the system of equations.

$$\begin{aligned}
 -\frac{3}{4}x + 20 &= \frac{4}{3}x - \frac{26}{3} \\
 \frac{25}{12}x &= \frac{86}{3} \\
 x &= \frac{344}{25} \\
 y &= \frac{4}{3}x - \frac{26}{3} \\
 y &= \frac{4}{3}\left(\frac{344}{25}\right) - \frac{26}{3} \\
 y &= \frac{242}{25}
 \end{aligned}$$

Use the distance formula to calculate the length of pipe required.

$$\begin{aligned}
 d_{AP} &= \sqrt{[13.76 - (-16)]^2 + (9.68 - 32)^2} \\
 &= \sqrt{885.6576 + 498.1824} \\
 &= \sqrt{1383.84} \\
 &= 37.2
 \end{aligned}$$

To calculate the length of pipe required to join house B, I need to write an equation for the pipe.

$$\begin{aligned}
 y &= -\frac{3}{4}x + b \\
 -24 &= -\frac{3}{4}(22) + b \\
 b &= -\frac{15}{2} \\
 y &= -\frac{3}{4}x - \frac{15}{2}
 \end{aligned}$$

To calculate the point of intersection, solve the system of equations.

$$\begin{aligned}
 -\frac{3}{4}x - \frac{15}{2} &= \frac{4}{3}x - \frac{26}{3} \\
 \frac{25}{12}x &= \frac{7}{6} \\
 x &= \frac{14}{25} \\
 y &= \frac{4}{3}x - \frac{26}{3} \\
 y &= \frac{4}{3}\left(\frac{14}{25}\right) - \frac{26}{3} \\
 y &= -\frac{198}{25}
 \end{aligned}$$

Use the distance formula to calculate the length of pipe required.

$$\begin{aligned}
 d_{BP} &= \sqrt{(0.56 - 22)^2 + [-7.92 - (-24)]^2} \\
 &= \sqrt{459.6736 + 258.5664} \\
 &= \sqrt{718.24} \\
 &= 26.8
 \end{aligned}$$

To calculate the length of pipe required to join house C, I need to write an equation for the pipe.

$$\begin{aligned}
 y &= -\frac{3}{4}x + b \\
 8 &= -\frac{3}{4}(56) + b \\
 b &= 50 \\
 y &= -\frac{3}{4}x + 50
 \end{aligned}$$

To calculate the point of intersection, solve the system of equations.

$$-\frac{3}{4}x + 50 = \frac{4}{3}x - \frac{26}{3}$$

$$\frac{25}{12}x = \frac{176}{3}$$

$$x = \frac{704}{25}$$

$$y = \frac{4}{3}x - \frac{26}{3}$$

$$y = \frac{4}{3}\left(\frac{704}{25}\right) - \frac{26}{3}$$

$$y = \frac{722}{25}$$

Use the distance formula to calculate the length of pipe required.

$$\begin{aligned} d_{OP} &= \sqrt{(28.16 - 56)^2 + (28.88 - 8)^2} \\ &= \sqrt{775.0656 + 435.9744} \\ &= \sqrt{1211.04} \\ &= 34.8 \end{aligned}$$

Assuming the connection charge is proportional to distance, A would have the highest charge.

b) To illuminate all three homes equally, the lamp should be located an equal distance from each home. Calculate the point of intersection of two right bisectors. First investigate the right bisector of AB.

$$m_{AB} = \frac{32 - (-24)}{-16 - 22}$$

$$= -\frac{56}{38}$$

$$= -\frac{28}{19}$$

$$m_{\text{bisector}} = \frac{19}{28}$$

$$M_{AB} = \left(\frac{-16 + 22}{2}, \frac{32 + (-24)}{2} \right)$$

$$= (3, 4)$$

The right bisector of BC is

$$m_{BC} = \frac{8 - (-24)}{56 - 22}$$

$$= \frac{32}{34}$$

$$= \frac{16}{17}$$

$$m_{\text{bisector}} = -\frac{17}{16}$$

$$M_{BC} = \left(\frac{22 + 56}{2}, \frac{-24 + 8}{2} \right)$$

$$= (39, -8)$$

The equation is

$$y = \frac{19}{28}x + b$$

$$4 = \frac{19}{28}(3) + b$$

$$b = \frac{55}{28}$$

$$y = \frac{19}{28}x + \frac{55}{28}$$

The equation is

$$y = -\frac{17}{16}x + b$$

$$-8 = -\frac{17}{16}(39) + b$$

$$b = \frac{535}{16}$$

$$y = -\frac{17}{16}x + \frac{535}{16}$$

Now calculate the point of intersection by solving the system of equations.

$$-\frac{17}{16}x + \frac{535}{16} = \frac{19}{28}x + \frac{55}{28} \quad y = \frac{19}{28}x + \frac{55}{28}$$

$$-\frac{195}{112}x = -\frac{3525}{112} \quad = \frac{19}{28}\left(\frac{235}{13}\right) + \frac{55}{28}$$

$$x = \frac{235}{13} \quad = \frac{185}{13}$$

The lamp should be placed at about (18.1, 14.2).

17. To classify the triangle, I will put each equation into $y = mx + b$ form. This will allow me to see if the triangle has a right angle.

Equation 1:

$$x + y = 11$$

$$y = -x + 11$$

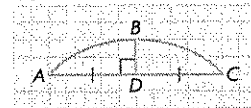
Equation 2:

$$x - y = 1$$

$$y = x - 1$$

This is a right triangle, since equation 1 and 2 are perpendicular, having slopes -1 and 1 .

18.



They could use the chord intersection rule to determine an E such that $AD \times DC = BD \times DE$, where $BD + DE = \text{diameter}$.

19. To determine the median, I would calculate the midpoint of the opposite side, then determine the slope of the segment between the vertex and the midpoint, which gives m , then substitute one of these points into the equation of a line to determine b . To determine the altitude, I would calculate the slope of the opposite side and determine its negative reciprocal, which gives m , then I would substitute the vertex into the equation of a line to determine b .

20. Let C represent the centroid. Begin by determining C .

$$M_{RP} = \left(\frac{1 + (-1)}{2}, \frac{2 + 2}{2} \right)$$

$$= (0, 2)$$

$$M_{QR} = \left(\frac{4 + 1}{2}, \frac{-4 + 2}{2} \right)$$

$$= \left(\frac{5}{2}, -1 \right)$$

$$M_{PQ} = \left(\frac{-1 + 4}{2}, \frac{2 + (-4)}{2} \right)$$

$$= \left(\frac{3}{2}, -1 \right)$$

Determine the equation for the median of R :

$$m_{\text{median } R} = \frac{-1 - 2}{\frac{3}{2} - 1}$$

$$= \frac{-3}{\frac{1}{2}}$$

$$= -\frac{3}{\frac{1}{2}}$$

$$= -6$$

$$y = -6x + b$$

$$2 = -6(1) + b$$

$$b = 8$$

The equation is

$$y = -6x + 8.$$

Determine the equation for the median of Q:

$$\begin{aligned} m_{\text{median } Q} &= \frac{2 - (-4)}{0 - 4} & y &= -\frac{3}{2}x + b \\ &= -\frac{6}{4} & 2 &= -\frac{3}{2}(0) + b \\ &= -\frac{3}{2} & b &= 2 \\ & & y &= -\frac{3}{2}x + 2 \end{aligned}$$

Now solve the system to determine that centroid.

$$-\frac{3}{2}x + 2 = -6x + 8$$

$$\frac{9}{2}x = 6$$

$$x = \frac{4}{3}$$

$$y = -6x + 8$$

$$y = -6\left(\frac{4}{3}\right) + 8$$

$$y = -8 + 8$$

$$y = 0$$

The centroid is $\left(\frac{4}{3}, 0\right)$.

Now determine the length of all the segments.

$$\begin{aligned} d_{PC} &= \sqrt{\left[\frac{4}{3} - (-1)\right]^2 + (0 - 2)^2} \\ &= \sqrt{\frac{49}{9} + 4} \\ &= \sqrt{\frac{85}{9}} \end{aligned}$$

$$\begin{aligned} d_{OC} &= \sqrt{\left(\frac{4}{3} - 4\right)^2 + [0 - (-4)]^2} \\ &= \sqrt{\frac{64}{9} + 16} \\ &= \sqrt{\frac{208}{9}} \end{aligned}$$

$$\begin{aligned} d_{RC} &= \sqrt{\left(\frac{4}{3} - 1\right)^2 + (0 - 2)^2} \\ &= \sqrt{\frac{1}{9} + 4} \\ &= \sqrt{\frac{37}{9}} \end{aligned}$$

$$M_{PQ} = \left(\frac{3}{2}, -1\right), M_{QR} = \left(\frac{5}{2}, -1\right), M_{RP} = (0, 2), C = \left(\frac{4}{3}, 0\right)$$

$$\begin{aligned} d_{CM_{QR}} &= \sqrt{\left(\frac{4}{3} - \frac{5}{2}\right)^2 + [0 - (-1)]^2} \\ &= \sqrt{\frac{49}{36} + 1} \\ &= \sqrt{\frac{85}{36}} \end{aligned}$$

$$\begin{aligned} d_{CM_{RP}} &= \sqrt{\left(\frac{4}{3} - 0\right)^2 + (0 - 2)^2} \\ &= \sqrt{\frac{16}{9} + 4} \\ &= \sqrt{\frac{52}{9}} \end{aligned}$$

$$\begin{aligned} d_{CM_{PQ}} &= \sqrt{\left(\frac{4}{3} - \frac{3}{2}\right)^2 + [0 - (-1)]^2} \\ &= \sqrt{\frac{1}{36} + 1} \\ &= \sqrt{\frac{37}{36}} \end{aligned}$$

$$\frac{PC}{CM_{QR}} = \frac{QC}{CM_{RP}} = \frac{RC}{CM_{PQ}} = 2$$

21. a) To prove these points are on the circle, substitute them both in and solve. Both sides should be equal.

$$\begin{aligned} x^2 + y^2 &= (3a)^2 + a^2 \\ &= 9a^2 + a^2 \\ &= 10a^2 \\ x^2 + y^2 &= a^2 + (-3a)^2 \\ &= a^2 + 9a^2 \\ &= 10a^2 \end{aligned}$$

b) Let C represent the centre: $C = (0, 0)$. I need to show the slopes of these segments are negative reciprocals of each other.

$$\begin{aligned} M_{RQ} &= \left(\frac{3a+a}{2}, \frac{a+(-3a)}{2}\right) & m_{RQ} &= \frac{a - (-3a)}{3a - a} \\ &= \left(\frac{4a}{2}, \frac{-2a}{2}\right) & &= \frac{4a}{2a} \\ &= (2a, -a) & &= 2 \\ m_{CM_{RQ}} &= \frac{-a - 0}{2a - 0} \\ &= -\frac{1}{2} \end{aligned}$$

The segments intersect at a right angle.

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$$\begin{aligned} 1. M &= \left(\frac{-25 + 40}{2}, \frac{20 + 36}{2}\right) \\ &= \left(\frac{15}{2}, \frac{56}{2}\right) \\ &= (7.5, 28) \end{aligned}$$

2. a)

$$M_{AC} = \left(\frac{-4+2}{2}, \frac{4+(-2)}{2} \right) \\ = (-1, 1)$$

Determine the slope of the line passing through $(-1, 1)$ and $(-4, -2)$.

$$m = \frac{1 - (-2)}{-1 - (-4)} \\ = \frac{3}{3} \\ = 1$$

b) If the median is also an altitude then it will have a slope which is the negative reciprocal of AC.

$$m_{AC} = \frac{4 - (-2)}{-4 - 2} \\ = -\frac{6}{6} \\ = -1$$

Since 1 and -1 are negative reciprocals, the median is also the altitude.

3. First calculate the slope and the midpoint of MN.

$$M_{MN} = \left(\frac{-5+2}{2}, \frac{2+(-2)}{2} \right) \\ = \left(\frac{-3}{2}, 0 \right)$$

$$m_{MN} = \frac{2 - (-2)}{-5 - 2} \\ = -\frac{4}{7}$$

Using the slope (1) and point $(-4, -2)$, determine the equation of the right bisector.

$$y = mx + b \\ y = x + b \\ -2 = (-4) + b \\ b = 2$$

The equation is $y = x + 2$.

The negative reciprocal slope is $\frac{7}{4}$.

Use the slope and the point to determine the equation of the bisector.

$$y = mx + b \\ y = \frac{7}{4}x + b \\ 0 = \frac{7}{4} \left(-\frac{3}{2} \right) + b \\ b = \frac{21}{8} \\ y = \frac{7}{4}x + \frac{21}{8}$$

4. To determine which point is closer to the origin, I will compare distances.

$$d_{PO} = \sqrt{(-24-0)^2 + (56-0)^2} \\ = \sqrt{576 + 3136} \\ = \sqrt{3712} \\ \approx 60.92$$

$$d_{QO} = \sqrt{(35-0)^2 + (-43-0)^2} \\ = \sqrt{1225 + 1849} \\ = \sqrt{3074} \\ \approx 55.44$$

The closer point is Q.

$$5. d = \sqrt{(105-20)^2 + (82-110)^2} \\ = \sqrt{7225 + 784} \\ = \sqrt{8009} \\ \approx 89.5$$

The builder would need about 89.5 units of cable.

6.

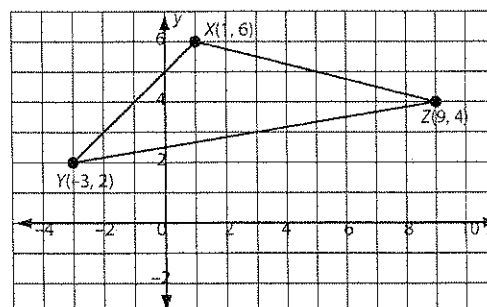
$$d_{QR} = \sqrt{[2 - (-3)]^2 + (6-1)^2} \quad d_{RS} = \sqrt{(-3-6)^2 + (1-2)^2} \\ = \sqrt{25 + 25} \quad = \sqrt{(-9)^2 + (-1)^2} \\ = \sqrt{50} \quad = \sqrt{81 + 1} \\ \approx 7.071 \quad = \sqrt{82} \\ \approx 9.055$$

$$d_{SQ} = \sqrt{(2-6)^2 + (6-2)^2} \\ = \sqrt{(-4)^2 + 4^2} \\ = \sqrt{16 + 16} \\ = \sqrt{32} \\ \approx 5.657$$

$$P = 7.071 + 9.055 + 5.657 \\ = 21.78$$

The perimeter is about 21.8 units.

7. I graphed the points. Inspecting the graph, the median from YZ to X is much shorter than the other two, so I just need to look at the other two to see which is longer.



Median from XZ to Y:

$$M_{XZ} = \left(\frac{1+9}{2}, \frac{6+4}{2} \right) \\ = (5, 5)$$

Use $(5, 5)$ and $(-3, 2)$ to determine the length of the median.

$$d = \sqrt{[5 - (-3)]^2 + (5-2)^2} \\ = \sqrt{8^2 + 3^2} \\ = \sqrt{64 + 9} \\ = \sqrt{73} \\ \approx 8.5$$

Median from XY to Z:

$$M_{XY} = \left(\frac{1+(-3)}{2}, \frac{6+2}{2} \right) \\ = (-1, 4)$$

Use $(-1, 4)$ and $(9, 4)$ to calculate the length of the median.

$$\begin{aligned}d &= \sqrt{(-1-9)^2 + (4-4)^2} \\&= \sqrt{(-10)^2 + 0^2} \\&= \sqrt{100+0} \\&= \sqrt{100} \\&= 10\end{aligned}$$

The longest median is 10 units.

8. a) $x^2 + y^2 = r^2$
 $(-8)^2 + 15^2 = r^2$
 $289 = r^2$

The equation is $x^2 + y^2 = 289$.

b) $x^2 + y^2 = 289$
 $x^2 + 0 = 289$
 $x = \pm 17$

The x-intercepts are $(17, 0)$ and $(-17, 0)$.

$$\begin{aligned}x^2 + y^2 &= 289 \\0 + y^2 &= 289 \\y &= \pm 17\end{aligned}$$

The y-intercepts are $(0, 17)$ and $(0, -17)$.

For the three points, I chose $(8, 15)$ and have different combinations of signs. We know any combination of signs with $(-8, 15)$ will be on the circle because any point 17 units away from the origin will be on the circle. Since the terms in the distance formula are squared, the sign change does not affect the distance from the origin. So, if $(-8, 15)$ is on the circle, any combination of signs will also be on the circle. I chose $(8, 15)$, since they were whole numbers though I could have chosen any x-value between -17 and 17 and solved for the y-value. The points are: $(8, 15)$, $(8, -15)$, $(-8, -15)$

9. Use the length of the diameter to calculate the radius.

$$\begin{aligned}d_{CD} &= \sqrt{[20 - (-20)]^2 + (-21 - 21)^2} & r &= \frac{58}{2} \\&= \sqrt{40^2 + (-42)^2} & &= 29 \\&= \sqrt{1600 + 1764} & r^2 &= 841 \\&= \sqrt{3364} \\&= 58\end{aligned}$$

The equation is $x^2 + y^2 = 841$.

10. The centre of the circle is at $(0, 0)$ and the radius in 4 units.

$$\begin{aligned}x^2 + y^2 &= 4^2 \\x^2 + y^2 &= 16\end{aligned}$$

11. $(-2)^2 + k^2 = 20$
 $4 + k^2 = 20$
 $k^2 = 16$
 $k = \pm 4$

12.

$$\begin{aligned}d_{AB} &= \sqrt{(1-2)^2 + (1-5)^2} & d_{BC} &= \sqrt{(2-6)^2 + (5-6)^2} \\&= \sqrt{1+16} & &= \sqrt{16+1} \\&= \sqrt{17} & &= \sqrt{17} \\&\approx 4.12 & &\approx 4.12\end{aligned}$$

$$\begin{aligned}d_{CA} &= \sqrt{(1-6)^2 + (1-6)^2} \\&= \sqrt{25+25} \\&= \sqrt{50} \\&\approx 7.07\end{aligned}$$

Since $AB = BC \neq CA$, the triangle is isosceles.

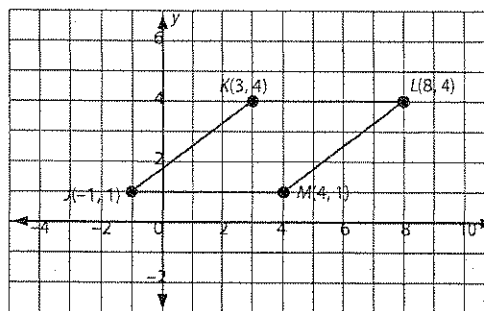
13. $d_{AB} = \sqrt{[1 - (-2)]^2 + [1 - (-1)]^2}$
 $= \sqrt{9+4}$
 $= \sqrt{13}$
 ≈ 3.61

$$\begin{aligned}d_{BC} &= \sqrt{(-2-3)^2 + [-1 - (-2)]^2} \\&= \sqrt{25+1} \\&= \sqrt{26} \\&\approx 5.10\end{aligned}$$

$$\begin{aligned}d_{CA} &= \sqrt{(1-3)^2 + [1 - (-2)]^2} \\&= \sqrt{4+9} \\&= \sqrt{13} \\&\approx 3.61\end{aligned}$$

Since $AB = CA \neq BC$, the triangle is isosceles.

14. I drew JKLM and could see that KL and JM are both horizontal lines 5 units long.



Since LM and KJ both join these segments, they must also be equal. Now to determine if $KL = JM = 5$ is equal to $LM = KJ$.

$$\begin{aligned}d_{LM} &= \sqrt{(8-4)^2 + (4-1)^2} \\&= \sqrt{16+9} \\&= \sqrt{25} \\&= 5\end{aligned}$$

Therefore, $JK = KL = LM = MJ = 5$ units and the quadrilateral is a rhombus.

15.

$$\begin{aligned}d_{RS} &= \sqrt{[-3 - (-1)]^2 + (2-6)^2} & d_{TU} &= \sqrt{(3-1)^2 + (5-1)^2} \\&= \sqrt{4+16} & &= \sqrt{4+16} \\&= \sqrt{20} & &= \sqrt{20} \\&\approx 4.47 & &\approx 4.47\end{aligned}$$

$$\begin{aligned}
 d_{ST} &= \sqrt{(-1-3)^2 + (6-5)^2} & d_{RU} &= \sqrt{(-3-1)^2 + (2-1)^2} \\
 &= \sqrt{16+1} & &= \sqrt{16+1} \\
 &= \sqrt{17} & &= \sqrt{17} \\
 &\approx 4.12 & &\approx 4.12
 \end{aligned}$$

The opposite sides are equal.

Now calculate the slopes of each side.

$$\begin{aligned}
 m_{RS} &= \frac{2-6}{-3-(-1)} & m_{TS} &= \frac{6-5}{-1-3} \\
 &= 2 & &= -\frac{1}{4} \\
 m_{TU} &= \frac{1-5}{1-3} & m_{RU} &= \frac{2-1}{-3-1} \\
 &= 2 & &= -\frac{1}{4}
 \end{aligned}$$

The opposite sides are equal and parallel but not perpendicular; so, the quadrilateral is a parallelogram.

$$\begin{aligned}
 16. M_{AB} &= \left(\frac{-3+(-5)}{2}, \frac{1+(-9)}{2} \right) \\
 &= (-4, -4)
 \end{aligned}$$

$$\begin{aligned}
 M_{BC} &= \left(\frac{-5+7}{2}, \frac{-9+(-1)}{2} \right) \\
 &= (1, -5)
 \end{aligned}$$

$$\begin{aligned}
 M_{CD} &= \left(\frac{7+3}{2}, \frac{-1+3}{2} \right) & M_{DA} &= \left(\frac{-3+3}{2}, \frac{1+3}{2} \right) \\
 &= (5, 1) & &= (0, 2)
 \end{aligned}$$

Now calculate the length of each new line segment.

$$\begin{aligned}
 d_{M_{AB}M_{DA}} &= \sqrt{(-4-0)^2 + (-4-2)^2} \\
 &= \sqrt{16+36} \\
 &= \sqrt{52} \\
 &\approx 7.21
 \end{aligned}$$

$$\begin{aligned}
 d_{M_{AB}M_{BC}} &= \sqrt{(-4-1)^2 + [-4-(-5)]^2} \\
 &= \sqrt{25+1} \\
 &= \sqrt{26} \\
 &\approx 5.10
 \end{aligned}$$

$$\begin{aligned}
 d_{M_{CD}M_{DA}} &= \sqrt{(5-0)^2 + (1-2)^2} \\
 &= \sqrt{25+1} \\
 &= \sqrt{26} \\
 &\approx 5.10
 \end{aligned}$$

$$\begin{aligned}
 d_{M_{BC}M_{CD}} &= \sqrt{(1-5)^2 + (-5-1)^2} \\
 &= \sqrt{16+36} \\
 &= \sqrt{52} \\
 &\approx 7.21
 \end{aligned}$$

Now calculate the slope of the midsegments.

$$\begin{aligned}
 m_{M_{AB}M_{BC}} &= \frac{-4-(-5)}{-4-1} & m_{M_{BC}M_{CD}} &= \frac{-5-1}{1-5} \\
 &= -\frac{1}{5} & &= \frac{3}{2} \\
 m_{M_{CD}M_{DA}} &= \frac{1-2}{5-0} & m_{M_{AB}M_{DA}} &= \frac{2-(-4)}{0-(-4)} \\
 &= -\frac{1}{5} & &= \frac{3}{2}
 \end{aligned}$$

The slopes of midsegments are $\frac{3}{2}$ and $-\frac{1}{5}$; opposite

sides are equal and parallel but not at 90° , therefore a parallelogram is formed.

17. A circle with centre $(5, -2)$ has equation

$$\begin{aligned}
 (x-5)^2 + (y+2)^2 &= r^2 \\
 r^2 &= (10-5)^2 + (10+2)^2 & r^2 &= (-7-5)^2 + (3+2)^2 \\
 r^2 &= 25 + 144 & r^2 &= 144 + 25 \\
 r^2 &= 169 & r^2 &= 169 \\
 r^2 &= (0-5)^2 + (-14+2)^2 \\
 r^2 &= 25 + 144 \\
 r^2 &= 169
 \end{aligned}$$

Since all of the equations have the same centre and the same radius, they must be the same circle.

18. a) To prove $\triangle PQR$ is a right angle triangle, show there are two sides with negative reciprocal slopes.

$$\begin{aligned}
 m_{PQ} &= \frac{7-2}{-2-(-4)} & m_{QR} &= \frac{2-(-2)}{-4-6} \\
 &= \frac{5}{2} & &= -\frac{2}{5}
 \end{aligned}$$

The slopes of PQ and QR are negative reciprocals, so PQ and QR form a right angle.

b)

$$\begin{aligned}
 M_{RP} &= \left(\frac{6+(-2)}{2}, \frac{-2+7}{2} \right) \\
 &= \left(2, \frac{5}{2} \right)
 \end{aligned}$$

$$\begin{aligned}
 d_{P_{mid}} &= \sqrt{[2-(-2)]^2 + (2.5-7)^2} \\
 &= \sqrt{16+20.25} \\
 &= \sqrt{36.25} \\
 &\approx 6.02
 \end{aligned}$$

$$\begin{aligned}
 d_{Q_{mid}} &= \sqrt{[2-(-4)]^2 + (2.5-2)^2} \\
 &= \sqrt{36+0.25} \\
 &= \sqrt{36.25} \\
 &\approx 6.02
 \end{aligned}$$

$$\begin{aligned}
 d_{R_{mid}} &= \sqrt{(2-6)^2 + [2.5-(-2)]^2} \\
 &= \sqrt{16+20.25} \\
 &= \sqrt{36.25} \\
 &\approx 6.02
 \end{aligned}$$

All the vertices are an equal distance away from the midpoint of the hypotenuse.

19. a) Insert the given points into the equation of a circle: $6^2 + 7^2 = r^2$ and $(-9)^2 + 2^2 = r^2$. If these points are on the same circle, their radius will be equal:

$$\begin{aligned} 6^2 + 7^2 &= (-9)^2 + 2^2 \\ 36 + 42 &= 81 + 4 \\ 85 &= 85 \end{aligned}$$

Then both points are the same distance from (0, 0) and must be on the circle.

b) Let A represent point (6, 7), let B represent point (-9, 2), let C represent the centre of the circle (0, 0), and let D represent the intersection of the line and the chord.

The new line must pass through this point:

$$\begin{aligned} M_{AB} &= \left(\frac{6 + (-9)}{2}, \frac{7 + 2}{2} \right) \\ &= \left(-\frac{3}{2}, \frac{9}{2} \right) \end{aligned}$$

Determine the equation of the new line and then verify that it crosses through the midpoint.

$$\begin{aligned} m_{AB} &= \frac{7 - 2}{6 - (-9)} \\ &= \frac{5}{15} \\ &= \frac{1}{3} \end{aligned}$$

$$m_{CD} = -3$$

Therefore the equation of the line through C is $y = -3x$.

Now I will let $x = -\frac{3}{2}$ and solve for y .

$$y = -3x$$

$$y = -3\left(-\frac{3}{2}\right)$$

$$y = \frac{9}{2}$$

Since the midpoint is the same as D , the line passes through the midpoint.

20. a) To prove the diagonals bisect each other, show their midpoints are the same point.

$$\begin{aligned} M_{JL} &= \left(\frac{5+6}{2}, \frac{-2+(-7)}{2} \right) & M_{KM} &= \left(\frac{3+8}{2}, \frac{-5+(-4)}{2} \right) \\ &= \left(\frac{11}{2}, -\frac{9}{2} \right) & &= \left(\frac{11}{2}, -\frac{9}{2} \right) \end{aligned}$$

$$M_{JL} = M_{KM}$$

b) Answers may vary, e.g., I conjecture that quadrilateral $JKLM$ is a square, since squares have diagonals that bisect each other.

c) Check the length of the sides and the slopes of the sides to determine if it is a rectangle, square, or rhombus.

Find the length of each side.

$$\begin{aligned} d_{JK} &= \sqrt{(5-8)^2 + [-2-(-4)]^2} \\ &= \sqrt{9+4} \\ &= \sqrt{13} \end{aligned}$$

$$\begin{aligned} d_{KL} &= \sqrt{(6-8)^2 + [-7-(-4)]^2} \\ &= \sqrt{4+9} \\ &= \sqrt{13} \end{aligned}$$

$$\begin{aligned} d_{LM} &= \sqrt{(6-3)^2 + [-7-(-5)]^2} \\ &= \sqrt{9+4} \\ &= \sqrt{13} \end{aligned}$$

$$\begin{aligned} d_{JM} &= \sqrt{(5-3)^2 + [-2-(-5)]^2} \\ &= \sqrt{4+9} \\ &= \sqrt{13} \end{aligned}$$

Find the slope of each line segment.

$$\begin{aligned} m_{JK} &= \frac{-2-(-4)}{5-8} & m_{KL} &= \frac{-4-(-7)}{8-6} \\ &= \frac{-2+4}{-3} & &= \frac{-4+7}{2} \\ &= -\frac{2}{3} & &= \frac{3}{2} \end{aligned}$$

$$\begin{aligned} m_{LM} &= \frac{-7-(-5)}{6-3} & m_{JM} &= \frac{-2-(-5)}{5-3} \\ &= \frac{-7+5}{3} & &= \frac{-2+5}{2} \\ &= -\frac{2}{3} & &= \frac{3}{2} \end{aligned}$$

Since $JK = KL = LM = MJ$, and $JK \perp KL$ and $LM \perp JM$, then the quadrilateral is a square.

21. First determine the equation for the altitude passing through R .

Use the slope and point R to determine the equation.

$$\begin{aligned} m_{PQ} &= \frac{-2-4}{0-4} \\ &= \frac{3}{2} \end{aligned}$$

$$m_R = -\frac{2}{3}$$

$$y = mx + b$$

$$5 = -\frac{2}{3}(-4) + b$$

$$b = 5 - \frac{8}{3}$$

$$b = \frac{7}{3}$$

$$y = -\frac{2}{3}x + \frac{7}{3}$$

Now, calculate the slope of the altitude passing through Q .

Use the slope and point Q to determine the equation.

$$\begin{aligned} m_{PR} &= \frac{-2-5}{0-(-4)} \\ &= -\frac{7}{4} \end{aligned}$$

$$m_R = \frac{4}{7}$$

$$y = mx + b$$

$$4 = \frac{4}{7}(4) + b$$

$$b = 4 - \frac{16}{7}$$

$$b = \frac{12}{7}$$

$$y = \frac{4}{7}x + \frac{12}{7}$$

To determine the point of intersection, set these two equations equal to each other.

$$\begin{aligned}\frac{4}{7}x + \frac{12}{7} &= -\frac{2}{3}x + \frac{7}{3} \\ \frac{26}{21}x &= \frac{13}{21} \\ x &= \frac{1}{2}\end{aligned}$$

Now calculate the y-value of the orthocentre.

$$\begin{aligned}y &= \frac{4}{7}x + \frac{12}{7} \\ y &= \frac{4}{7}\left(\frac{1}{2}\right) + \frac{12}{7} \\ y &= \frac{14}{7} \\ y &= 2 \\ \text{The orthocentre is} & (0.5, 2).\end{aligned}$$

22. Determine the equation of the median that passes through y.

$$\begin{aligned}M_{xz} &= \left(\frac{0+3}{2}, \frac{1+6}{2}\right) \\ &= \left(\frac{3}{2}, \frac{7}{2}\right)\end{aligned}$$

The equation between the midpoint and y is

$$\begin{aligned}m_y &= \frac{3.5 - (-1)}{1.5 - 6} \\ &= -1 \\ y &= mx + b \\ -1 &= -1(6) + b \\ b &= 5 \\ y &= -x + 5\end{aligned}$$

Determine the equation of the median that passes through Z.

$$\begin{aligned}M_{xy} &= \left(\frac{0+6}{2}, \frac{1+(-1)}{2}\right) \\ &= (3, 0)\end{aligned}$$

The equation between the midpoint and Z is

$$\begin{aligned}m_z &= \frac{0 - 6}{3 - 3} \\ &= \frac{6}{0} \\ &= \text{undefined}\end{aligned}$$

The slope is undefined, so the line is vertical and the equation is $x = 3$. To calculate the point of intersection, substitute in $x = 3$.

$$\begin{aligned}y &= -x + 5 \\ y &= -(3) + 5 \\ y &= 2\end{aligned}$$

The centroid is (3, 2).

23. To be an equal distance from three ranger stations, the towers needs to be built at the circumference of the triangle formed by the stations. The circumference is where the right bisectors of the triangle meet. Determine the equations of the three right bisectors.

For the right bisector of AB:

$$\begin{aligned}M_{AB} &= \left(\frac{-90+0}{2}, \frac{28+(-35)}{2}\right) \\ &= (-45, -3.5) \\ m_{AB} &= \frac{28 - (-35)}{-90 - 0} \\ &= -\frac{63}{90} \\ &= -\frac{7}{10} \\ m_{\text{bisector}} &= \frac{10}{7}\end{aligned}$$

The equation is:

$$\begin{aligned}-3.5 &= \frac{10}{7}(-45) + b \\ b &= \frac{851}{14} \\ y &= \frac{10}{7}x + \frac{851}{14}\end{aligned}$$

Now determine where they intersect by letting the two equations equal each other and solving.

$$\begin{aligned}-\frac{25}{11}x + \frac{1480}{11} &= \frac{10}{7}x + \frac{851}{14} \\ -\frac{285}{77}x &= -\frac{11359}{154} \\ x &= 19.9\end{aligned}$$

The new tower should be at about (19.9, 89.3).

24. Answers may vary, e.g., it will be a parallelogram because the slopes of the lines form two pairs, which are not negative reciprocals.

Solve each equation to determine their slopes.

Line 1:

$$\begin{aligned}3x + y - 4 &= 0 \\ y &= -3x + 4\end{aligned}$$

Line 2:

$$\begin{aligned}4x - 5y + 30 &= 0 \\ 4x + 30 &= 5y \\ \frac{4}{5}x + 6 &= y\end{aligned}$$

Line 3:

$$y = 3x - 1$$

Line 4:

$$\begin{aligned}-4x + 5y + 10 &= 0 \\ 5y &= 4x - 10 \\ y &= \frac{4}{5}x - 2\end{aligned}$$

For the right bisector of BC:

$$\begin{aligned}M_{BC} &= \left(\frac{125+0}{2}, \frac{20+(-35)}{2}\right) \\ &= (62.5, -7.5) \\ m_{BC} &= \frac{20 - (-35)}{125 - 0} \\ &= \frac{55}{125} \\ &= \frac{11}{25} \\ m_{\text{bisector}} &= -\frac{25}{11}\end{aligned}$$

The equation is:

$$\begin{aligned}-7.5 &= -\frac{25}{11}(62.5) + b \\ b &= \frac{1480}{11} \\ y &= -\frac{25}{11}x + \frac{1480}{11}\end{aligned}$$

Solve for the y-value:

$$\begin{aligned}y &= -\frac{25}{11}x + \frac{1480}{11} \\ y &= -\frac{25}{11}(19.93) + \frac{1480}{11} \\ y &= 89.3\end{aligned}$$

Since I have confirmed they are in parallel pairs, the shape must be a parallelogram.

25. a) The builder will want to connect the cable at the point where the two cables will be perpendicular. When the cables are perpendicular it is the shortest distance. I will determine the equation of the main power line and determine its perpendicular slope.

$$m_{TU} = \frac{5-12}{1-29} = \frac{-7}{-28} = \frac{1}{4}$$

$$y = mx + b$$

$$5 = \frac{1}{4}(1) + b$$

$$b = \frac{19}{4}$$

$$y = \frac{1}{4}x + \frac{19}{4}$$

$$m_{S \rightarrow TU} = -4$$

Now I know a point on the new cable and the slope, I can determine its equation.

$$y = mx + b$$

$$18 = -4(25) + b$$

$$b = 118$$

The equation is $y = -4x + 118$.

To determine the point where these power lines will be joined, I will calculate their intersection.

$$\frac{1}{4}x + \frac{19}{4} = -4x + 118$$

$$\frac{17}{4}x = \frac{453}{4}$$

$$x = \frac{453}{17}$$

I will determine the y-value.

$$y = -4x + 118$$

$$y = -4\left(\frac{453}{17}\right) + 118$$

$$y = \frac{194}{17}$$

The cables should be joined at $\left(\frac{453}{17}, \frac{194}{17}\right)$.

b) To calculate the length of the cable, I will use the distance formula and the two end points.

$$d = \sqrt{\left(\frac{453}{17} - 25\right)^2 + \left(\frac{194}{17} - 18\right)^2}$$

$$= \sqrt{2.713 + 43.405}$$

$$= \sqrt{46.118}$$

$$\approx 6.79$$

The builder will need about 6.79 m of cable.

Self-Test, page 126

1. a) $d_{AB} = \sqrt{(-6-14)^2 + [23-(-12)]^2}$

$$= \sqrt{(-20)^2 + 35^2}$$

$$= \sqrt{400 + 1225}$$

$$= \sqrt{1625}$$

$$\approx 40.3$$

A cable of about 40.3 m will be needed.

b) $M = \left(\frac{-6+14}{2}, \frac{23+(-12)}{2}\right)$

$$= \left(\frac{8}{2}, \frac{11}{2}\right)$$

$$= \left(4, \frac{11}{2}\right)$$

2. a) After 3 s, the radius will be $3 \times 12 = 36$.

$$x^2 + y^2 = r^2$$

$$x^2 + y^2 = 36^2$$

$$x^2 + y^2 = 1296$$

The equation is

$$x^2 + y^2 = 1296.$$

Now that I know $r = 60$, I can calculate the number of intervals of 12 there are. $60 \div 12 = 5$. The ripples will reach the bulrush after 5 s.

3. To prove the triangle is isosceles, show that exactly two sides have the same length.

$$d_{AB} = \sqrt{[1-(-3)]^2 + [2-(-1)]^2}$$

$$= \sqrt{4^2 + 3^2}$$

$$= \sqrt{16 + 9}$$

$$= \sqrt{25}$$

$$= 5$$

$$d_{BC} = \sqrt{(-3-0)^2 + [-1-(-5)]^2}$$

$$= \sqrt{(-3)^2 + 4^2}$$

$$= \sqrt{9 + 16}$$

$$= \sqrt{25}$$

$$= 5$$

$$d_{AC} = \sqrt{(1-0)^2 + [2-(-5)]^2}$$

$$= \sqrt{1^2 + 7^2}$$

$$= \sqrt{1 + 49}$$

$$\approx 7.07$$

$$AB = BC = 5$$

Now prove the triangle contains a right angle. I can do this by showing that there are two slopes, which are negative reciprocals of each other.

$$m_{AB} = \frac{-1-2}{-3-1} = \frac{-3}{-4} = \frac{3}{4}$$

$$m_{BC} = \frac{-1-(-5)}{-3-0} = \frac{-4}{-3} = \frac{4}{3}$$

$$m_{AB} = -\frac{1}{m_{BC}}$$

4. a) To prove PQRS is a rectangle, I will determine the slopes of all the sides. If opposite sides have the same slope and adjacent side are perpendicular then the figure is a rectangle.

$$m_{PQ} = \frac{39-(-13)}{-39-(-78)} = \frac{52}{39} = \frac{4}{3}$$

$$m_{QR} = \frac{-13-(-91)}{-78-26} = \frac{78}{104} = \frac{3}{4}$$

$$m_{PQ} = m_{RS}$$

$$= -\frac{1}{m_{QR}}$$

$$= -\frac{1}{m_{SP}}$$

$$m_{RS} = \frac{-91 - (-39)}{26 - 65} = \frac{52}{-39} = -\frac{4}{3}$$

$$m_{SP} = \frac{-39 - 39}{65 - (-39)} = \frac{78}{104} = \frac{3}{4}$$

The building lot is a rectangle.

b) The building lot is a rectangle, so I need to determine the length of only two sides.

$$d_{PQ} = \sqrt{[-39 - (-78)]^2 + [39 - (-13)]^2}$$

$$= \sqrt{39^2 + 52^2}$$

$$= \sqrt{1521 + 2704}$$

$$= \sqrt{4225}$$

$$= 65$$

$$d_{QR} = \sqrt{(-78 - 26)^2 + [-13 - (-91)]^2}$$

$$= \sqrt{(-104)^2 + 78^2}$$

$$= \sqrt{10816 + 6084}$$

$$= 130$$

$$P = 2(65 + 130)$$

$$P = 2(195)$$

$$P = 390$$

The perimeter of the building lot is 390 units.

5. If the diagonals perpendicularly bisect each other then the shape is a rhombus.

$$m_{JL} = \frac{-2 - 4}{2 - 2} = \frac{-6}{0}$$

$$M_{JL} = \left(\frac{2+2}{2}, \frac{4+(-2)}{2} \right) = (2, 1)$$

$$m_{KM} = \frac{1 - 1}{-2 - 6} = \frac{0}{-8}$$

$$M_{KM} = \left(\frac{6+(-2)}{2}, \frac{1+1}{2} \right) = (2, 1)$$

This shape is a rhombus.

6. To determine the location, determine the equation of the right bisector of each side and then determine where they intersect.

Begin with the right bisector of AB.

$$M_{AB} = \left(\frac{-24 + 56}{2}, \frac{16 + (-16)}{2} \right) = (16, 0)$$

$$m_{AB} = \frac{16 - (-16)}{-24 - 56} = -\frac{32}{80} = -\frac{2}{5}$$

Use (16, 0) and the reciprocal slope to determine the equation of the right bisector of AB.

$$y = mx + b$$

$$y = \frac{5}{2}x + b$$

$$0 = \frac{5}{2}(16) + b$$

$$b = -40$$

$$y = -\frac{5}{2}x - 40$$

Now, determine the right bisector of AC.

$$M_{AC} = \left(\frac{-24 + (-72)}{2}, \frac{16 + (-32)}{2} \right) = (-48, -8)$$

$$m_{AC} = \frac{16 - (-32)}{-24 - (-72)} = \frac{48}{48} = 1$$

Use (-48, -8) and the reciprocal slope to determine the equation of the right bisector of AC.

$$y = mx + b$$

$$y = -x + b$$

$$-8 = 48 + b$$

$$b = -56$$

The equation is $y = -x - 56$.

Now solve the system of equations.

$$-x - 56 = \frac{5}{2}x - 40$$

$$-\frac{7}{2}x = 16$$

$$x = -4.5714$$

Solve for y.

$$y = -(-4.5714) - 56$$

$$y = -51.4286$$

The fountain should be placed at about (-4.6, -51.4).

7. Answers may vary, e.g., calculate the slope of a line through the first two points, then substitute one of these points into the equation of a line to solve for the intercept. Substitute the third point into the equation of a line with a slope equal to the negative reciprocal of the first slope to determine the equation of the new line. Set these two equations equal to each other to calculate the intersection. Then calculate the distance between this intersection and the third point.

8. The three equations in $y = mx + b$ form are $y = 2x - 4$, $y = -0.5x + 1$, and $y = -3x - 9$. Since equation 1 and equation 2 have slopes that are negative reciprocals of each other, the triangle has a right angle. This also means the triangle cannot be equilateral. Determine the vertices of the triangle by finding the points of intersection of the lines.

2 × equation 1 = equation 2:

$$2x + 4y - 4 = 2x - y - 4$$

$$5y = 0$$

$$y = 0$$

Now, calculate the x-value using equation 1.

$$y = 2x - 4$$

$$0 = 2x - 4$$

$$2x = 4$$

$$x = 2$$

Let A be the vertex at (2, 0).

equation 2 + equation 3:

$$5x + 5 = 0$$

$$5x = -5$$

$$x = -1$$

Now, calculate the y-value using equation 3.

$$y = -3x - 9$$

$$y = -3(-1) - 9$$

$$y = 3 - 9$$

$$y = -6$$

Let B be the vertex at (-1, -6).

equation 3 = 3 × equation 1

$$3x + y + 9 = 3x + 6y - 6$$

$$15 = 5y$$

$$y = 3$$

Now, determine the x-value using equation 3.

$$\begin{aligned}y &= -3x - 9 \\3 &= -3x - 9 \\12 &= -3x \\x &= -4\end{aligned}$$

Let C be the vertex at $(-4, 3)$.

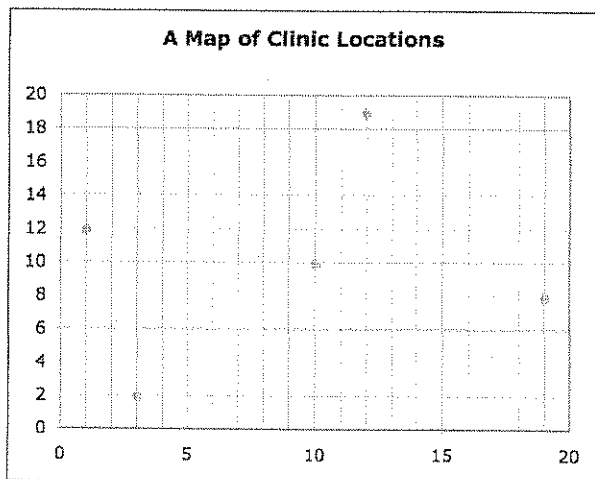
Now calculate the lengths of the sides.

$$\begin{aligned}d_{AB} &= \sqrt{(-1-2)^2 + (-6-0)^2} & d_{AC} &= \sqrt{[2-(-4)]^2 + (0-3)^2} \\&= \sqrt{9+36} & &= \sqrt{36+9} \\&= \sqrt{45} & &= \sqrt{45}\end{aligned}$$

Two sides are equal so it must be a right isosceles triangle.

Chapter Task, page 127

A.



The diagnostic centre should go around point $(10, 10)$ since this appears to be in the centre of all four clinics.

B. To determine the point which is the same distance from each clinic I will calculate the intersections of the perpendicular bisectors. First I will look at the perpendicular bisector of AB.

$$\begin{aligned}M_{AB} &= \left(\frac{1+12}{2}, \frac{12+19}{2} \right) & m_{AB} &= \frac{19-12}{12-1} \\&= (6.5, 15.5) & &= \frac{7}{11} \\m_{\text{bisector}} &= -\frac{11}{7}\end{aligned}$$

The equation of the perpendicular bisector through AB is

$$\begin{aligned}y &= mx + b \\15.5 &= -\frac{11}{7}(6.5) + b \\ \frac{217}{14} + \frac{143}{14} &= b \\ \frac{180}{7} &= b \\ y &= -\frac{11}{7}x + \frac{180}{7}\end{aligned}$$

For the perpendicular bisector of BC:

$$\begin{aligned}M_{BC} &= \left(\frac{19+12}{2}, \frac{8+19}{2} \right) & m_{BC} &= \frac{19-8}{12-19} \\&= (15.5, 13.5) & &= -\frac{11}{7} \\m_{\text{bisector}} &= \frac{7}{11}\end{aligned}$$

The equation of the perpendicular bisector through BC is

$$\begin{aligned}y &= mx + b \\13.5 &= \frac{7}{11}(15.5) + b \\ b &= \frac{297}{22} - \frac{217}{22} \\ b &= \frac{40}{11} \\ y &= \frac{7}{11}x + \frac{40}{11}\end{aligned}$$

Solve the system of equations.

$$\begin{aligned}\frac{7}{11}x + \frac{40}{11} &= -\frac{11}{7}x + \frac{180}{7} \\ 49x + 280 &= -121x + 1980 \\ 170x &= 1700 \\ x &= 10\end{aligned}$$

The y-coordinate is:

$$\begin{aligned}y &= \frac{7}{11}x + \frac{40}{11} \\ y &= \frac{7}{11}(10) + \frac{40}{11} \\ y &= 10\end{aligned}$$

The clinic should be located at point $(10, 10)$.

C.

$$\begin{aligned}d_{OP} &= \sqrt{(3-10)^2 + (2-10)^2} \\&= \sqrt{(-7)^2 + (-8)^2} \\&= \sqrt{49+64} \\&= \sqrt{113} \\&\approx 10.6\end{aligned}$$

D. From part B, the equation of the perpendicular

bisector of AB is $y = -\frac{11}{7}x + \frac{180}{7}$. Now look at the right bisector of BD.

$$\begin{aligned}M_{BD} &= \left(\frac{3+12}{2}, \frac{2+19}{2} \right) & m_{BD} &= \frac{19-2}{12-3} \\&= (7.5, 10.5) & &= \frac{17}{9} \\m_{\text{bisector}} &= \frac{9}{17}\end{aligned}$$

The equation of the right bisector through BD is

$$\begin{aligned}y &= mx + b \\10.5 &= \frac{9}{17}(7.5) + b \\ b &= \frac{357}{34} - \frac{135}{34} \\ b &= \frac{111}{17} \\ y &= \frac{9}{17}x + \frac{111}{17}\end{aligned}$$

Solve the system of equations.

$$\begin{aligned}\frac{9}{17}x + \frac{111}{17} &= -\frac{11}{7}x + \frac{180}{7} \\ \frac{250}{119}x &= \frac{2283}{119} \\ x &= 9.132\end{aligned}$$

The y-coordinate is:

$$\begin{aligned}y &= \frac{9}{17}x + \frac{180}{7} \\ y &= \frac{9}{17}(9.132) + \frac{111}{17} \\ y &= 11.364\end{aligned}$$

The clinic should be located at point $(9.132, 11.364)$.

$$\begin{aligned}
 \text{E. } d_{CP} &= \sqrt{(19 - 9.132)^2 + (8 - 11.364)^2} \\
 &= \sqrt{9.868^2 + (-3.364)^2} \\
 &= \sqrt{97.377 + 11.316} \\
 &= \sqrt{108.693} \\
 &\approx 10.43
 \end{aligned}$$

F. For points A, C, and D:

I will now look at the right bisector of CD.

$$\begin{aligned}
 M_{CD} &= \left(\frac{3+19}{2}, \frac{2+8}{2} \right) \\
 &= (11, 5) \\
 m_{CD} &= \frac{8-2}{19-3} \\
 &= \frac{6}{16} \\
 &= \frac{3}{8}
 \end{aligned}$$

$$m_{\text{bisector}} = -\frac{8}{3}$$

The equation of the right bisector through CD is

$$\begin{aligned}
 y &= mx + b \\
 5 &= -\frac{8}{3}(11) + b \\
 b &= \frac{15}{3} + \frac{88}{3} \\
 b &= \frac{103}{3} \\
 y &= -\frac{8}{3}x + \frac{103}{3}
 \end{aligned}$$

For the right bisector of AD:

$$\begin{aligned}
 M_{AD} &= \left(\frac{3+1}{2}, \frac{2+12}{2} \right) \\
 &= (2, 7) \\
 m_{AD} &= \frac{12-2}{1-3} \\
 &= -\frac{10}{2} \\
 &= -5
 \end{aligned}$$

$$m_{\text{bisector}} = \frac{1}{5}$$

The equation of the right bisector through AD is

$$\begin{aligned}
 y &= mx + b \\
 7 &= \frac{1}{5}(2) + b \\
 b &= \frac{35}{5} - \frac{2}{5} \\
 b &= \frac{33}{5} \\
 y &= \frac{1}{5}x + \frac{33}{5}
 \end{aligned}$$

Solve the system of equations.

$$\begin{aligned}
 -\frac{8}{3}x + \frac{103}{3} &= \frac{1}{5}x + \frac{33}{5} \\
 \frac{43}{15}x &= \frac{416}{15} \\
 x &\approx 9.674
 \end{aligned}$$

The y-coordinate is

$$\begin{aligned}
 y &= \frac{1}{5}x + \frac{33}{5} \\
 y &= \frac{1}{5}(9.674) + \frac{33}{5} \\
 y &\approx 8.535
 \end{aligned}$$

The clinic should be located at point (9.673, 8.535).

$$\begin{aligned}
 d_{BP} &= \sqrt{(12 - 9.674)^2 + (19 - 8.535)^2} \\
 &= \sqrt{2.326^2 + 10.465^2} \\
 &= \sqrt{5.410 + 109.516} \\
 &= \sqrt{114.926} \\
 &\approx 10.72
 \end{aligned}$$

For points B, C, and D:

From part B, the equation of the right bisector of BC is

$$y = \frac{7}{11}x + \frac{40}{11}$$

From part D, the equation of the right bisector of BD is

$$y = \frac{9}{17}x + \frac{111}{17}$$

Solve the system of equations.

$$\begin{aligned}
 \frac{9}{17}x + \frac{111}{17} &= \frac{7}{11}x + \frac{40}{11} \\
 \frac{20}{187}x &= \frac{541}{187} \\
 x &\approx 27.05
 \end{aligned}$$

The y-coordinate is

$$\begin{aligned}
 y &= \frac{9}{17}x + \frac{111}{17} \\
 y &= \frac{9}{17}(27.05) + \frac{111}{17} \\
 y &\approx 20.85
 \end{aligned}$$

The point of the clinic should be (27.05, 20.85).

$$\begin{aligned}
 d_{AP} &= \sqrt{(1 - 27.05)^2 + (12 - 20.85)^2} \\
 &= \sqrt{(-26.05)^2 + (-8.85)^2} \\
 &= \sqrt{678.60 + 78.32} \\
 &= \sqrt{756.92} \\
 &\approx 27.51
 \end{aligned}$$

G. I would choose (10, 10) as a location for the clinic since all but one of the points are very close or at this location.